Imperial College London

Computational Neurodynamics

Topic 4
Simple Neuron Models

Pedro Mediano

(Slides: Pedro Mediano & Murray Shanahan)

Overview

- Low-dimensional neuron models
- Integrate-and-fire neurons
- Izhikevich's model

1D Neuron Models 1

- Our goal is to simulate large numbers of interacting neurons. Therefore, we should make simulations computationally cheap.
- Much research exists on *low-dimensional* (approximate) neuron models. In the 1D case, they often take the form:

$$\tau \frac{dv}{dt} = f(v) + I$$

for some function *f*, where τ is the neuron's timescale and I is the dendritic (input) current.

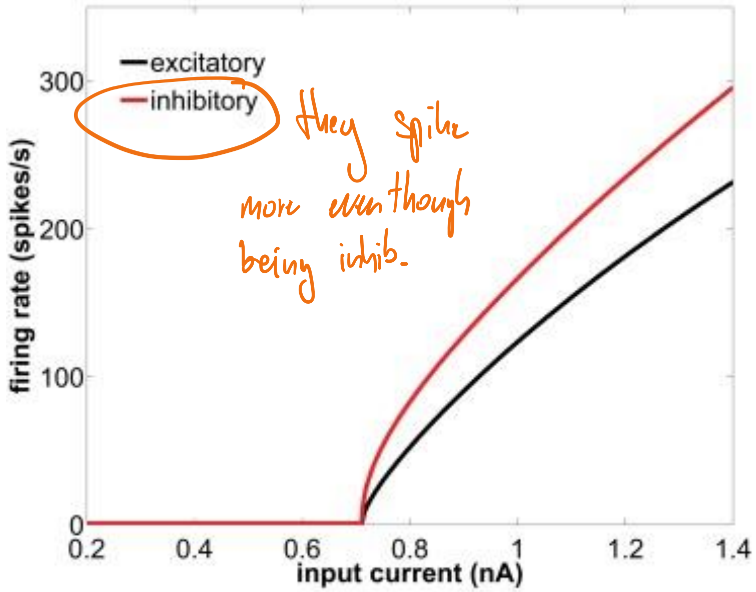
- Often these are expanded with a fixed voltage threshold θ that triggers a spike and reset.

1D Neuron Models 2

- For a given current *I*, the solution to the following equation shows the steady-state voltage v_s:

$$\tau \left.\frac{dv}{dt}\right|_{v_s} = 0$$

- There is a critical current I_c that makes $v_s = \theta$. The neuron will begin firing for any current higher than I_c.
- Plotting input current vs firing rate reveals the neuron's *frequency-intensity* curve.



Integrate-and-Fire Neurons 1

- The Hodgkin-Huxley model is biologically accurate, but computationally expensive
- At the opposite end of the spectrum we have the *leaky integrate-and-fire* (LIF) model, which is computationally inexpensive but has limited biological fidelity
- In the LIF model, the membrane potential v is given by

$$\tau \frac{dv}{dt} = v_r - v + RI$$

the cell potential

→ why v_r? If we dont feel current, the „v" will converge to „v_r"

input current

where v_r is the resting potential, I is the dendritic current, and τ and R are constants. (We'll use $\tau = 5$, $R = 1$, and $v_r = -65$mV)

Integrate-and-Fire Neurons 2

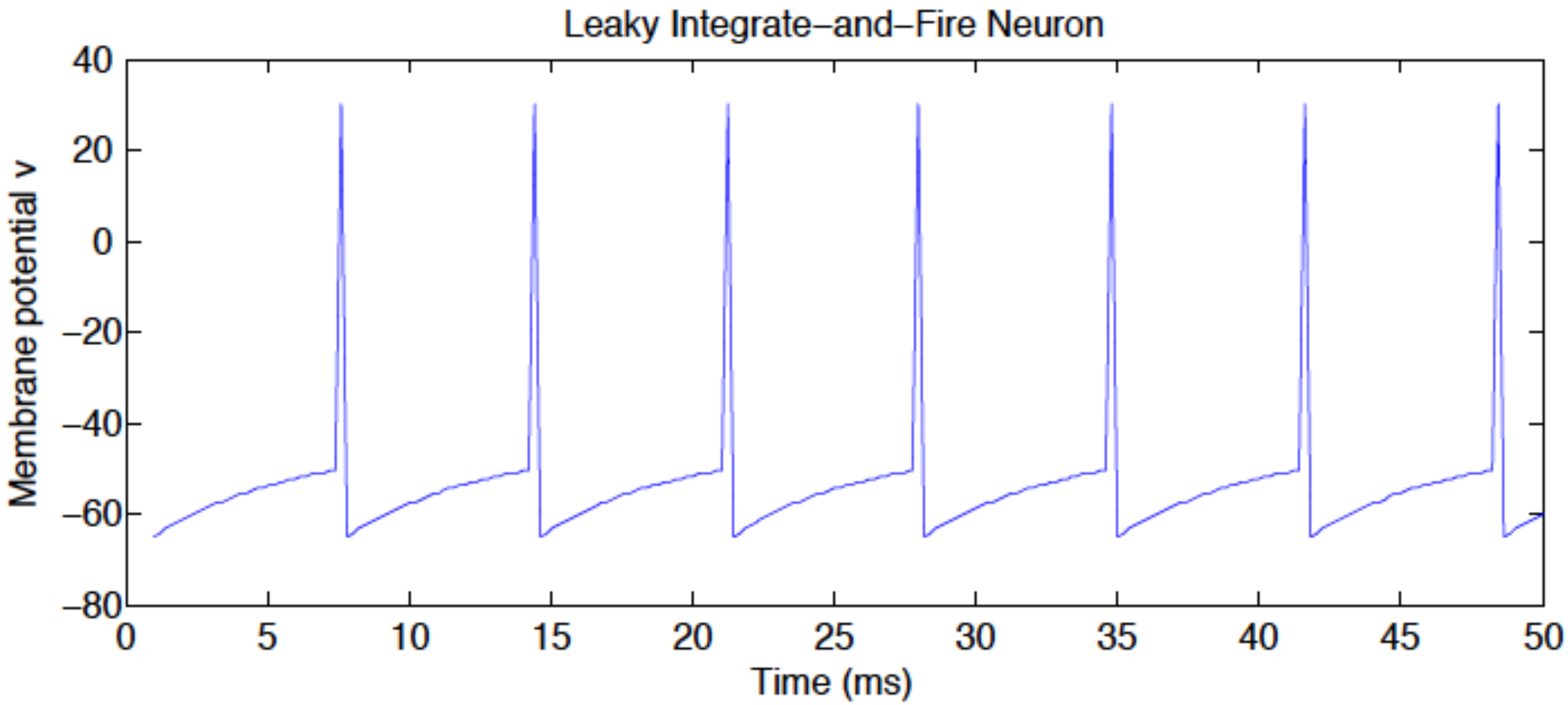
- The sub-threshold (before spiking) dynamics of the sodium and potassium currents are approximated by the $v_r - v$ term
- The detailed dynamics of the spike itself are ignored. Instead, when the membrane potential reaches a threshold, we record a spike and explicitly reset the neuron

$$\text{if } v \geq \vartheta \text{ then } v \leftarrow v_r$$

- A good value for the threshold ϑ is –50mV
- An instantaneous value to represent the actual spike (a Dirac pulse) can be inserted immediately before the neuron is reset

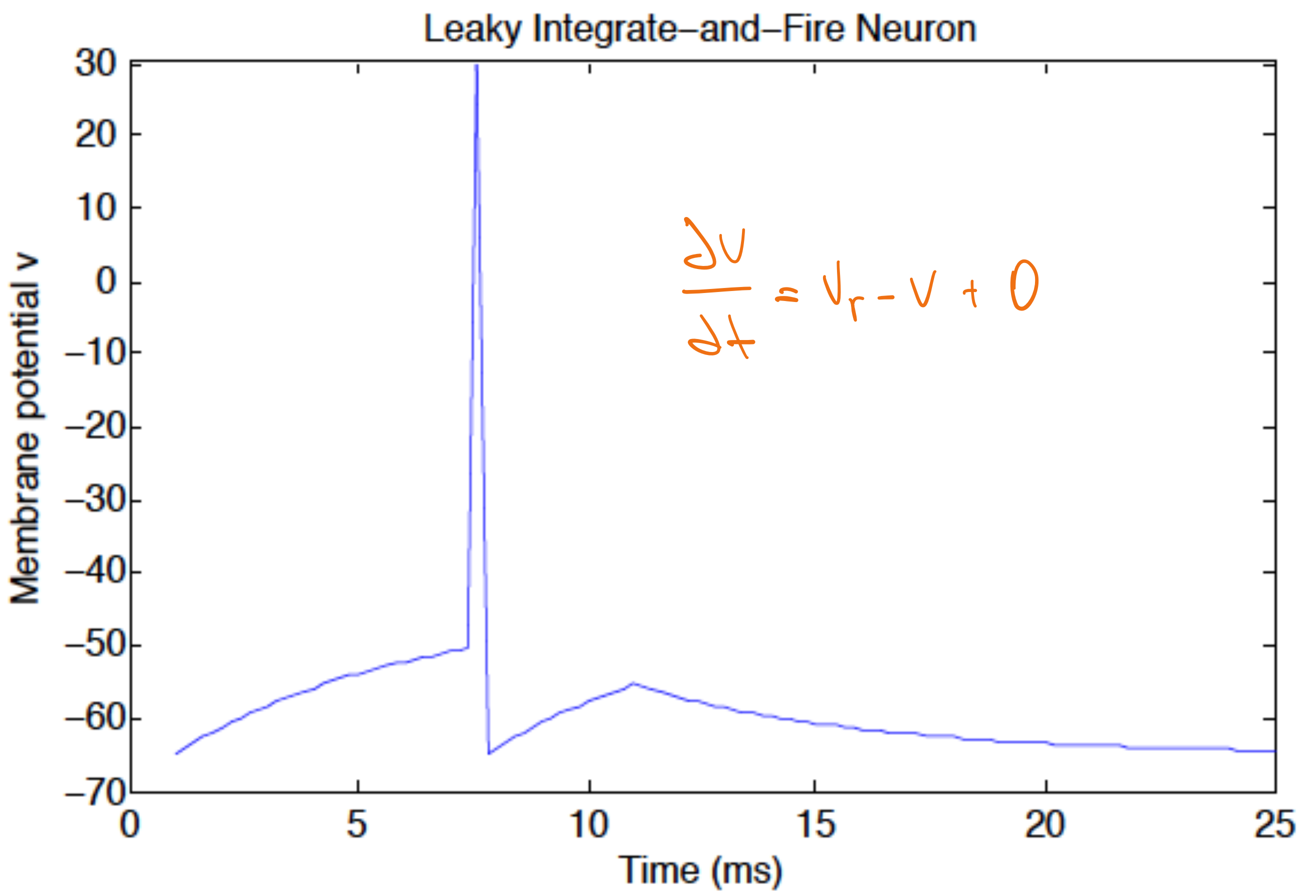
Integrate-and-Fire Neurons 3

- Here we see the regular spiking behaviour of a leaky integrate-and-fire neuron, simulated using the Euler method, and subject to a constant dendritic current $I = 20$



LIF Leakage

- In the absence of dendritic current, the membrane potential drifts back down to its resting value, thanks to the leakage current
- Here, the dendritic current is shut off after 10ms



LIF Refractory Periods

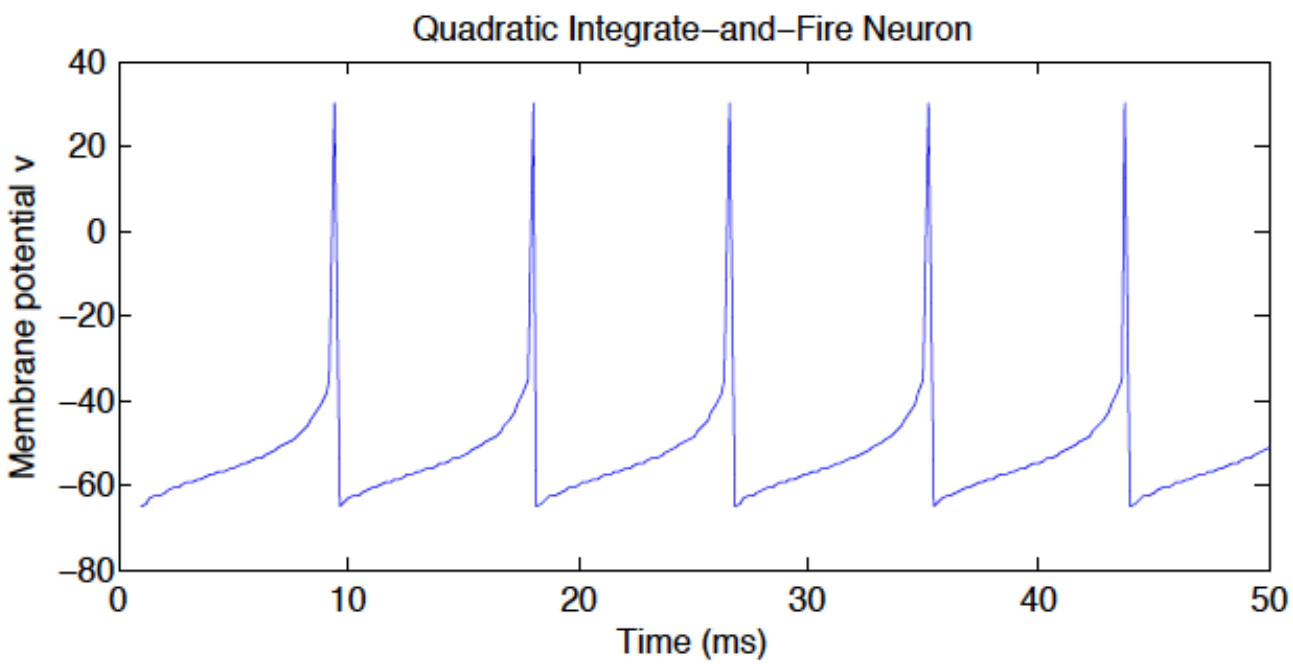
- The simple integrate-and-fire model has no refractory period — the period during which a real neuron is unable to fire even if it receives high dendritic current
- To overcome this, we can force the neuron to rest by introducing an *absolute refractory period* α
- We simply adjust the conditions under which a spike occurs to take account of the time since the last spike
- Let t_{spike} be the time of the most recent spike. Then we have

$$\text{if } v \geq \vartheta \text{ and } t - t_{spike} > \alpha \text{ then } \begin{cases} v \leftarrow v_r \\ t_{spike} \leftarrow t \end{cases}$$

Quadratic Integrate-and-Fire

- The sub-threshold profile of the membrane potential is modeled more accurately in the quadratic integrate-and-fire model

$$\tau \frac{dv}{dt} = a(v_r - v)(v_c - v) + RI$$



- In the absence of dendritic current, v decays to the resting potential v_r, as long as it is below a critical value v_c
- But if it is above v_c it increases quickly until the neuron fires

Izhikevich Neurons 1

- Integrate-and-fire neurons have a limited repertoire of signalling behaviours compared to the variety foud in real neurons, but they are computationally inexpensive to simulate
- Hodgkin-Huxley neurons are biologically accurate, but computationally expensive
- Izhikevich neurons (introduced by Eugene Izhikevich in 2003) are a good compromise between computational efficiency and a biologically realistic repertoire of behaviours
- This is the main neuron model used on this course

Izhikevich Neurons 2

- In Izhikevich's model, the membrane potential v and a recovery variable u are governed by two equations

$$\frac{dv}{dt} = 0.04v^2 + 5v + 140 - u + I$$

quadratic int-and-fire

this prevents neuron to spike in certain situations

$$\frac{du}{dt} = a(bv - u)$$

where I is the dendritic current, and a and b are parameters of the model

- Note that, without the recovery variable, the model is equivalent to QIF

– in total having 4 params.

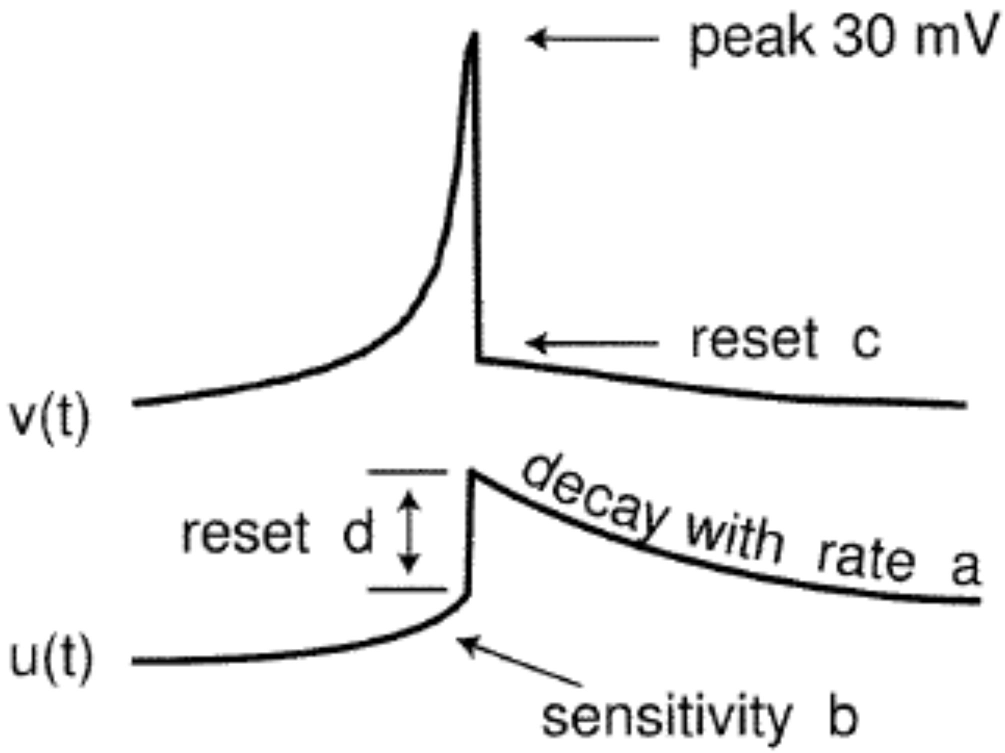
Izhikevich Neurons 3

- A spike occurs, and the neuron is reset, when the membrane potential reaches a threshold (30mV)

$$\text{if } v \geq 30 \text{ then } \begin{cases} v \leftarrow c \\ u \leftarrow u + d \end{cases}$$

- By varying the four parameters of the model a, b, c, and d, a wide variety of realistic signalling behaviours can be obtained
- Izhikevich neurons will be used throughout this course. Two types of neurons will be used — regular spiking (excitatory) and fast spiking (inhibitory) — by setting a, b, c, and d appropriately

Izhikevich Parameters



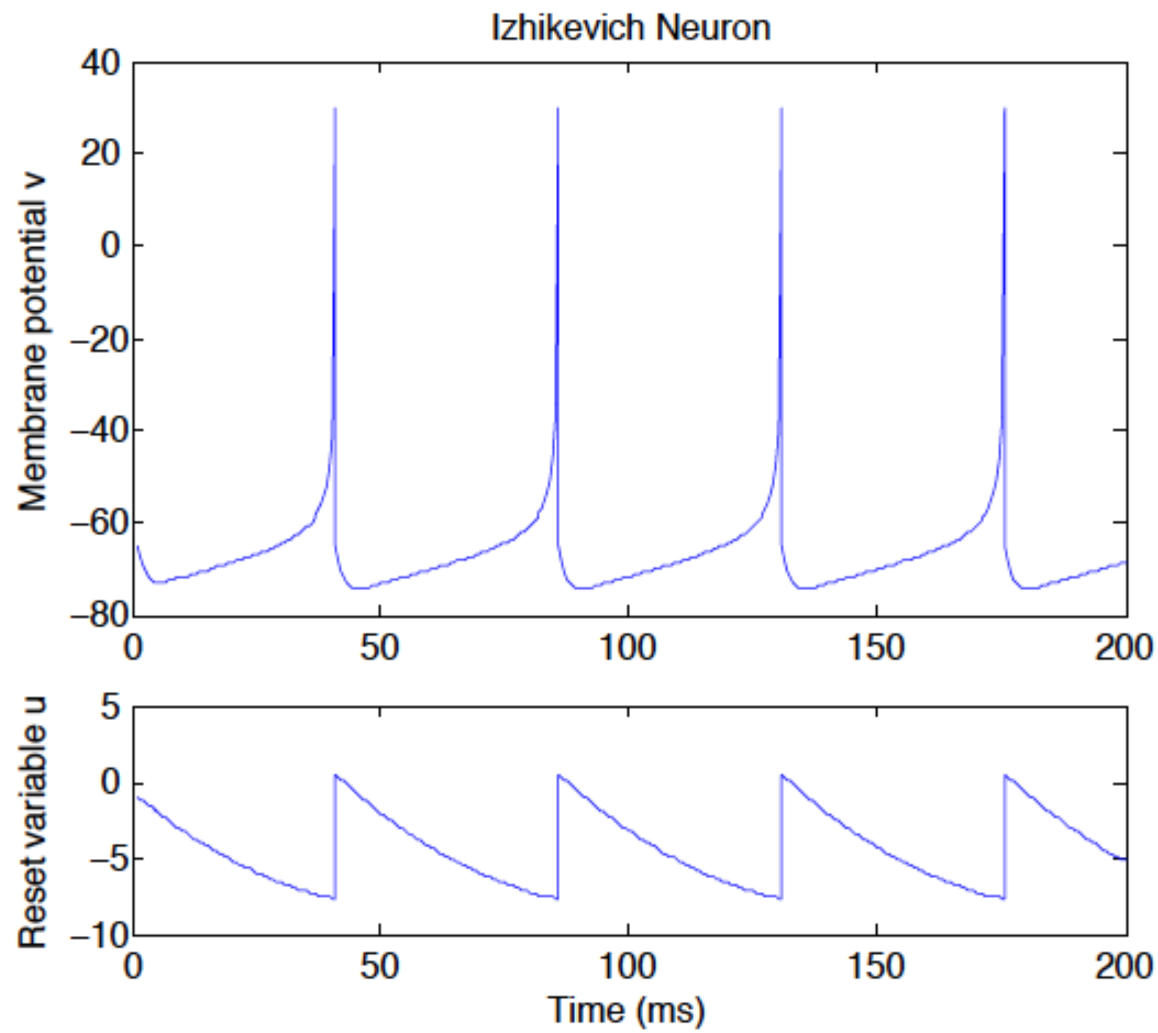
From Izhikevich, 2003

- This figure (from Izhikevich's paper) shows the role of each of the four parameters of the model
- Remember that a high value for u slows the rate of increase of v, and makes it harder for the neuron to fire

Excitatory Izhikevich Neurons

- If we set a = 0.02, b = 0.2, c = –65, and d = 8, we get *regular spiking* behaviour
- This is suitable for modelling *excitatory neurons*

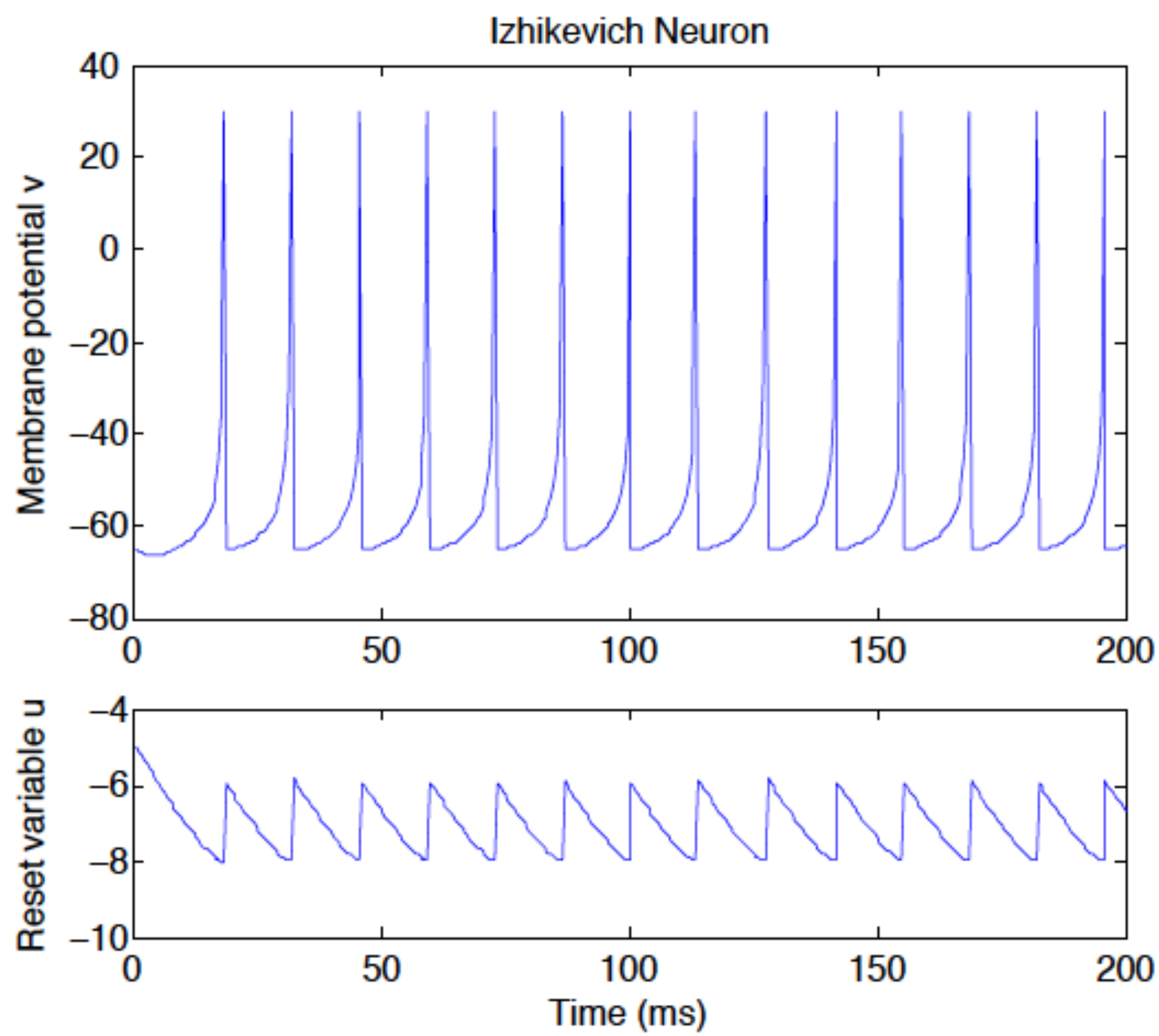
We model refractory period, however only „soft", so we still can fire, but sometimes it needs a lot more power



Inhibitory Izhikevich Neurons

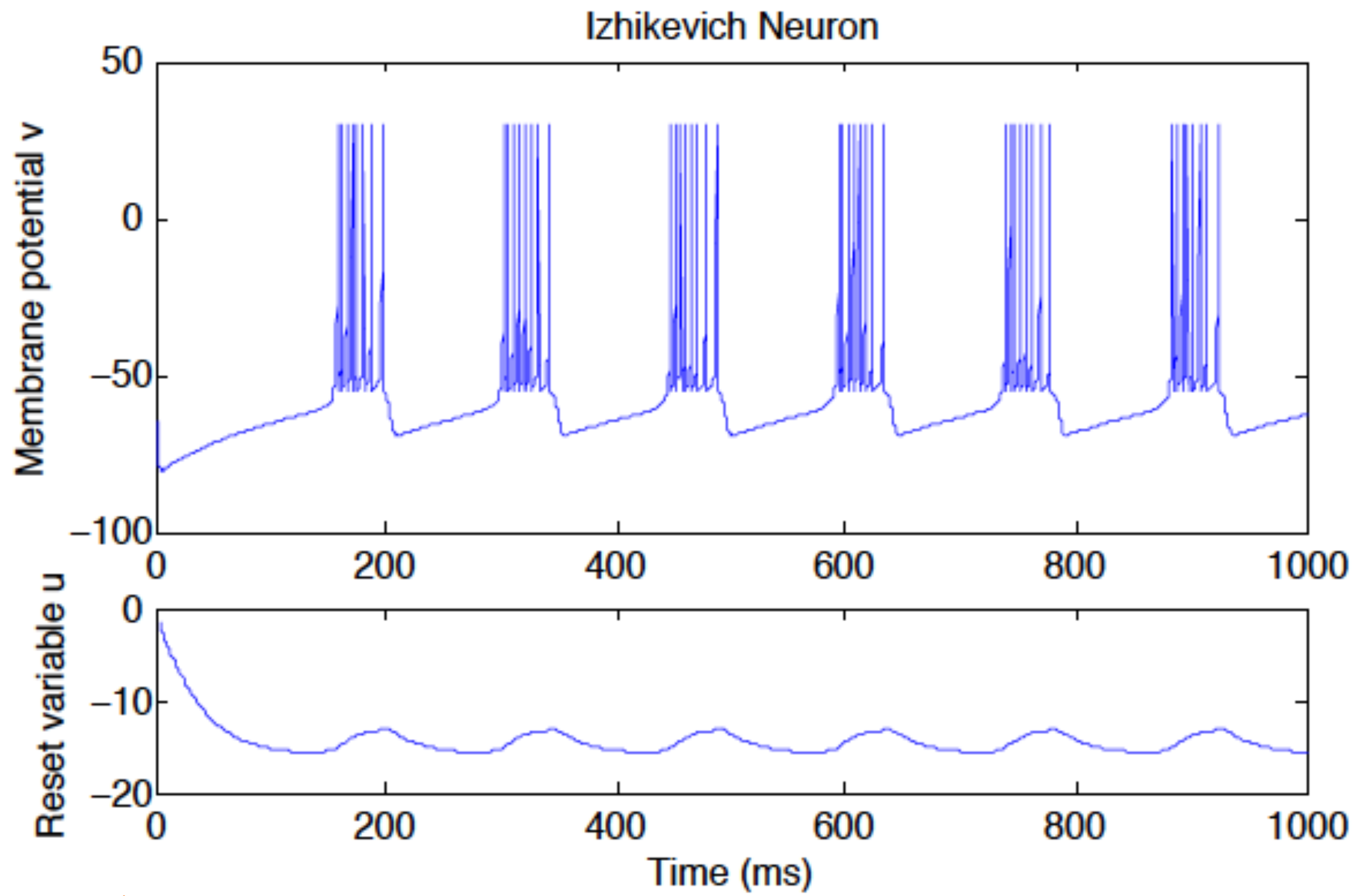
- If we set a = 0.02, b = 0.25, c = –65, and d = 2, we get *fast spiking* behaviour
- This is suitable for modelling *inhibitory neurons*

Inhibitory neurons are spiking generally more frequently



Bursting Izhikevich Neurons

- If we set a = 0.02, b = 0.2, c = –50, and d = 2, we get *bursting* behaviour
- This is gives rise to oscillations in the theta band (4-7 Hz), typical of hippocampus



– this shows that we can simulate a lot of different behaviours

Other Neuron Models

- There are several other common neuron models in addition to those we've looked at
  - FitzHugh-Nagumo
  - Hindmarsh-Rose
  - Morris-Lecar
- But we will use the Izhikevich model for the rest of this course, because it presents a good compromise between biological fidelity and computational efficiency

Related Reading

Izhikevich, E. (2003). Simple Model of Spiking Neurons. *IEEE Transactions on Neural Networks* 14 (6), 1569–1572.

Izhikevich, E. (2007). *Dynamical Systems in Neuroscience*. MIT Press. (Chapters 1, 8)