

Pro jaké $a \in \mathbb{R}$ je matice $\begin{pmatrix} a & 1 & 0 \\ 1 & a & 1 \\ 0 & 1 & a \end{pmatrix}$ pozitivně definitní?

$$\begin{vmatrix} a & 1 & 0 \\ 1 & a & 1 \\ 0 & 1 & a \end{vmatrix} = a^3 - 2a$$

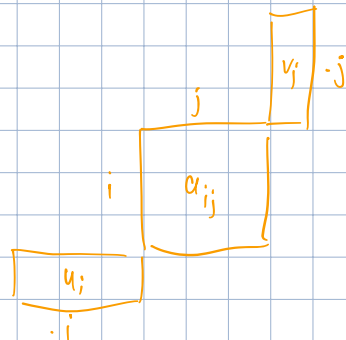
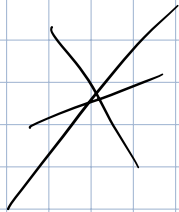
$$a > 0$$

$$(a-1) \cdot (a+1) > 0$$

$$a \in (-\infty, -1) \cup (1, \infty)$$

$$\begin{vmatrix} a & 1 \\ 1 & a \end{vmatrix} = a^2 - 1$$

$$|a| = a$$



$$A = \begin{pmatrix} 1 & -2 & 0 \\ 2 & 0 & -1 \\ -2 & 2 & 0 \end{pmatrix} \text{ je matice bilineární formy na } \mathbb{K}^3$$

Uřete analyticky vjadřem této formy:

$$= \sum_{i=1}^n \sum_{j=1}^n a_{ij} u_i v_j = u^T A v$$

$$\mathbb{K} = \mathbb{R}$$

$$u_1 v_1 + 2u_2 v_1 - 2u_3 v_1 - 2u_1 v_2 + 2u_3 v_2 - u_1 v_3$$

$$g(u) = f(u, u) = u_1^2 - 2u_1 u_3 + u_2 u_3 \rightarrow 0 \quad A' = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 0 & \frac{1}{2} \\ -1 & \frac{1}{2} & 0 \end{pmatrix}$$

$$g(u) = u^T A u$$

$$f(u, v) = u^T A v$$

$$\mathbb{K} = \mathbb{Z}_2$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$f((u_1, u_2, u_3)^T, (v_1, v_2, v_3)^T) = u_1 v_1 + u_2 v_3$$

$$A' \text{ neexistuje, protože } \frac{1}{2} \in \mathbb{Z}_2 = 0$$

Rozhodněte, zdali platí $g(u) > 0 \quad \forall$ netriviální $u = (x_1, x_2, x_3) \in \mathbb{R}^3$

Tedy jestli je pozitivně definitní!

$$a) g(u) = x_1^2 + 2x_1 x_2 + 2x_1 x_3 + 2x_2^2 + 5x_3^2$$

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 0 \\ 1 & 0 & 5 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & -1 & 4 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 3 \end{pmatrix}$$

$$b) g(u) = x_1^2 + 2x_1 x_2 + 2x_1 x_3 + 2x_2^2 + 5x_3^2 + 2a x_2 x_3$$

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & a \\ 1 & a & 5 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & a-1 \\ 0 & a-1 & 4 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & a-1 \\ 0 & 0 & 5(a-1)(a-1) \end{pmatrix}$$

$$4 - (a-1)^2 > 0$$

$$(a-1)^2 < 4$$

$$a \in (-1, 3)$$

Forma g má vzhľadom ku kon. bázi 4-ovl. vyjádrení:

$$g(u) = 2x^2 + 2xy - y^2 - 2yt - t^2, \text{ kde } u = (x, y, z, t)^T.$$

Najdite vyjádrení včoi bázi $X = \{(1,1,1,1)^T, (1,1,1,0)^T, (1,1,0,0)^T, (1,0,0,0)^T\}$

$$A = \begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 \end{pmatrix}$$

$$[id]_{x_h} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$[u]_h = (4, 4, 4, 3)^T$$

$$[u]_x \rightsquigarrow [u]_h = [id]_h [u]_x$$

$$[u]_x \cdot [id]_h = [u]_h$$

!! Prvok mezi bázi !!

$$g(u) = ([id]_{x_h} [u]_x)^T A [id]_{x_h} \cdot [u]_x$$

$$= [u]_x \cdot [id]_{x_h} \cdot A \cdot [id]_{x_h} \cdot [u]_x$$

$\uparrow$
B $\rightarrow$ matice g včoi x