$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 4 & 1 \\ 2 & 3 & 3 \end{vmatrix} = (-1)^{2+1} \cdot 4 \cdot \begin{vmatrix} 2 & 3 \\ 3 & 3 \end{vmatrix} +$$

$$(-1)^{2+2} \cdot 4 \cdot \begin{vmatrix} 1 & 3 \\ 2 & 3 \end{vmatrix} +$$

$$(-1)^{2+3} \cdot 1 \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = 2 + 3 + 1 = 1$$

nad $\mathbb{Z}_5$

podle 2. řádku

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 4 & 1 \\ 2 & 3 & 3 \end{vmatrix} = +3 \cdot \begin{vmatrix} 4 & 4 \\ 2 & 3 \end{vmatrix} - 1 \cdot \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} + 3 \cdot \begin{vmatrix} 1 & 2 \\ 4 & 4 \end{vmatrix} = 2+1+3 = 1$$

podle 3. sloupce

$$\begin{matrix} 1 & 2 & 0 & 1 & 2 \\ 5 & 6 & 0 & 5 & 6 \\ 0 & 0 & 8 & 0 & 0 \end{matrix} \qquad \begin{matrix} 0 & 1 & 2 & 0 & 1 \\ 0 & 5 & 6 & 0 & 5 \\ 7 & 0 & 0 & 7 & 0 \end{matrix}$$

$$\to \begin{vmatrix} 0 & 1 & 2 & 0 \\ 3 & 0 & 0 & 4 \\ 0 & 5 & 6 & 0 \\ 7 & 0 & 0 & 8 \end{vmatrix} = -3 \cdot \begin{vmatrix} 1 & 2 & 0 \\ 5 & 6 & 0 \\ 0 & 0 & 8 \end{vmatrix} + 4 \cdot \begin{vmatrix} 0 & 1 & 2 \\ 0 & 5 & 6 \\ 7 & 0 & 0 \end{vmatrix} = -16$$

$48 - 80$

$-32$

$+96$

$42 - 70$

$-28 \cdot 4 = -112$

(4)

$$\begin{vmatrix} a_1+x & a_2 & \cdots & a_n \\ a_1 & a_2+x & \cdots & a_n \\ \vdots & & \ddots & \vdots \\ a_n & \cdots & & a_n+x \end{vmatrix} = \begin{vmatrix} x & 0 & \cdots & 0 & -x \\ 0 & x & \cdots & 0 & -x \\ & & \ddots & & \vdots \\ & & & x & \\ a_1 & a_2 & \cdots & \cdots & a_n+x \end{vmatrix} =$$

$$= \begin{vmatrix} x & & & 0 \\ & \ddots & & \\ & & \ddots & \\ a_1 & \cdots & \cdots & a_{n-1} \end{vmatrix} = x^{n-1} \cdot \left( x + \sum_{i=1}^{n} a_i \right)$$

6) $17 \mid 697$ & $17 \mid 476$ & $17 \mid 969$

$$17 \left| \begin{vmatrix} 6 & 9 & 7 \\ 4 & 7 & 6 \\ 9 & 6 & 9 \end{vmatrix} \right. \Rightarrow \begin{vmatrix} 6 & 9 & 7 \\ 4 & 7 & 6 \\ 9 & 6 & 9 \end{vmatrix} = \begin{vmatrix} 6 & 9 & 697 \\ 4 & 7 & 476 \\ 9 & 6 & 969 \end{vmatrix} =$$

$100x \quad 10x \quad x$

$$= 17 \cdot \begin{vmatrix} 6 & 9 & \frac{697}{17} \\ 4 & 7 & \frac{476}{17} \\ 9 & 6 & \frac{969}{17} \end{vmatrix} \Rightarrow \in \mathbb{Z} \Rightarrow \det \in \mathbb{Z}$$

8)
$$\begin{pmatrix} 1 & 2 & -2 & 2 \\ 2 & 3 & 1 & 1 \\ -1 & 2 & 3 & -1 \\ 1 & -1 & 1 & -1 \end{pmatrix} X = \begin{pmatrix} -2 \\ 2 \\ -1 \\ 4 \end{pmatrix}$$

$$x_i = \frac{|A_{i\to b}|}{|A|}$$

$$|A| = \ldots = 6$$

$$|A_{1\to b}| = \begin{vmatrix} -2 & 2 & -2 & 2 \\ 2 & 3 & 1 & 1 \\ -1 & 2 & 3 & -1 \\ 4 & -1 & 1 & -1 \end{vmatrix} = \ldots = 12 \qquad x_1 = \frac{12}{6} = 2$$

$$|A_{2\to b}| = \begin{vmatrix} 1 & -2 & -2 & 2 \\ 2 & 2 & 1 & 1 \\ -1 & -1 & 3 & -1 \\ 1 & 4 & 1 & -1 \end{vmatrix} = \ldots = -6 \qquad x_2 = \frac{-6}{6} = -1$$

$\vdots \qquad \vdots \qquad \vdots$