$$\begin{vmatrix} 0 & x & y & z \\ x & 0 & z & y \\ y & z & 0 & x \\ z & y & x & 0 \end{vmatrix} = \begin{vmatrix} 0 & x & y & z \\ 1 & 0 & \frac{z}{x} & \frac{y}{x} \\ 1 & \frac{z}{y} & 0 & \frac{x}{y} \\ 1 & \frac{y}{z} & \frac{x}{z} & 0 \end{vmatrix} \cdot xyz = \begin{vmatrix} 0 & xyz & xyz & xyz \\ 1 & 0 & z^2 & y^2 \\ 1 & z^2 & 0 & x^2 \\ 1 & y^2 & x^2 & 0 \end{vmatrix} \cdot \frac{xyz}{zx \cdot zy \cdot xy}$$

$$\cdot zy \quad \cdot zx \quad \cdot xy$$

$$\frac{xyz}{zx \cdot zy \cdot xy} = \frac{1}{xyz}$$

$x, y, z \neq 0$

$$\begin{vmatrix} 0 & xyz & xyz & xyz \\ 1 & 0 & z^2 & y^2 \\ 1 & z^2 & 0 & x^2 \\ 1 & y^2 & x^2 & 0 \end{vmatrix} \cdot \frac{1}{xyz} = \begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & z^2 & y^2 \\ 1 & z^2 & 0 & x^2 \\ 1 & y^2 & x^2 & 0 \end{vmatrix}$$

$1/xyz$

Obecně:

$$p(x,y,z) \longrightarrow p(x,y,z)\,\frac{xyz}{xyz} \longrightarrow p(x,y,z)$$

toto není definované!!

2)

$$A = \begin{pmatrix} 1 & 0 & 1 \\ -2 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \qquad \operatorname{adj}(A) = \begin{pmatrix} 1 & +1 & -1 \\ +2 & 1 & -2 \\ -2 & -1 & 1 \end{pmatrix}$$

$$\operatorname{adj}(A)_{\mathbb{Z}_5} = \begin{pmatrix} 1 & 1 & 4 \\ 2 & 1 & 3 \\ 3 & 4 & 1 \end{pmatrix}$$

$\mathbb{R}$

$$|A| = -1 \qquad A^{-1} = \begin{pmatrix} -1 & -1 & 1 \\ -2 & -1 & 2 \\ 2 & 1 & -2 \end{pmatrix}$$

$\mathbb{Z}_5$

$$|A| = 4 \qquad A^{-1} = \begin{pmatrix} 4 & 4 & 1 \\ 3 & 4 & 2 \\ 2 & 1 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 2 \\ 1 & 2 & 0 \\ 0 & 1 & 1 \end{pmatrix} \qquad \operatorname{adj}(A) = \begin{pmatrix} 2 & 2 & -4 \\ -1 & 1 & 2 \\ 1 & -1 & 2 \end{pmatrix} \qquad \mathbb{Z}_5 \quad \operatorname{adj}(A) = \begin{pmatrix} 2 & 2 & 1 \\ 4 & 1 & 2 \\ 1 & 4 & 2 \end{pmatrix}$$

③ $a(3,1,1)^T,\ b = (2,1,1)^T,\ c = (2,3,2)^T$

$$\begin{vmatrix} 3 & 2 & 2 \\ 1 & 1 & 3 \\ 1 & 1 & 2 \end{vmatrix} = 1 \nearrow$$ tzn. že tento rovnoběžnostěn má objem 1.

④ $f: \mathbb{R}^3 \to \mathbb{R}^3$

Determinant reprezentuje míru zvětšení/zmenšení objemu.

$$\begin{matrix} (1,3,1)^T & & (3,1,0)^T \\ (1,0,3)^T & \to & (1,0,2)^T \\ (1,1,1)^T & & (4,1,5)^T \end{matrix}$$

$$V(A) = \tfrac{4}{3}\pi, \quad V(f(A)) = |\det([f])| \cdot V(A) = \pi$$

$[f] \cdot A = B$

$$[f] = B \cdot A^{-1} = \frac{\det(B)}{\det(A)}$$

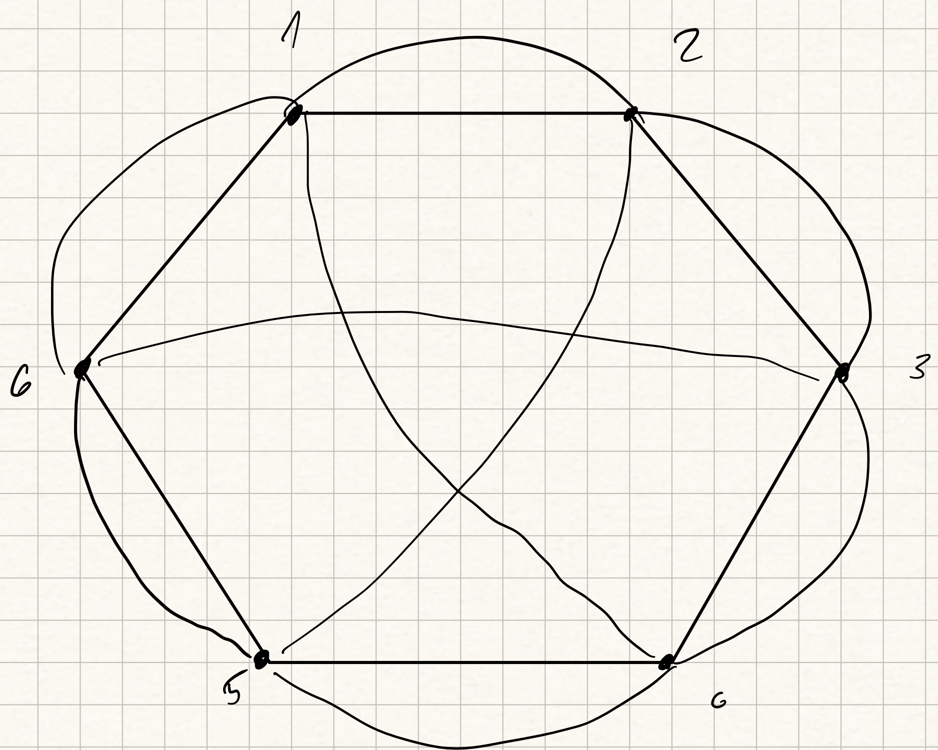
$\det(AB) = \det(A) \cdot \det(B)$

$\det(A^{-1}) = \dfrac{1}{\det(A)}$

$$\begin{vmatrix} 1 & 1 & 1 \\ 3 & 0 & 1 \\ 1 & 3 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & -3 & -2 \\ 0 & 2 & 0 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & -1 & -2 \\ 0 & 2 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 0 & 0 & -2 \\ 0 & 2 & 0 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & -2 \end{vmatrix} = \underline{\underline{4}}$$

$$\begin{vmatrix} 3 & 1 & 4 \\ 1 & 0 & 1 \\ 0 & 2 & 5 \end{vmatrix} = \underline{\underline{-3}}$$



$$H(6) = \det\left(L_6^{1,1}\right) = 960$$

$$\begin{pmatrix} 5 & -2 & 0 & -1 & 0 & -2 \\ -2 & 5 & -2 & 0 & -1 & 0 \\ 0 & -2 & 5 & -2 & 0 & -1 \\ -1 & 0 & -2 & 5 & -2 & 0 \\ 0 & -1 & 0 & -2 & 5 & -2 \\ -2 & 0 & -1 & 0 & -5 & 5 \end{pmatrix}$$

(7) $\mathbb{Z}_5$

$$p(x) = 4x^{20} + 3x^{17} + 2x^{16} + x^{13} + 3x^{12} + 2x^{10} + 4x^{9} + 2x^{7} + 2x^{5} + x + 3.$$

$\forall a \in \mathbb{Z}_p$: $a^{p-1} = 1$
$a \neq 0$

$a^{p} = a$

$x^{p} = x$

$$p^1(x) = a_4 \cdot x^4 + a_3 \cdot x^3 + a_2 \cdot x^2 + a_1 \cdot x^1 + a_0$$

$a_0 = 3$

$a_1 = 3 + 1 + 4 + 2 + 1 = 1$

$a_2 = 2$

$a_3 = 2$

$a_4 = 4 + 2 + 3 = 4$

$p'(x) = 4x^4 + 2x^3 + 2x^2 + x + 3$

$a^{p-1} = 1$

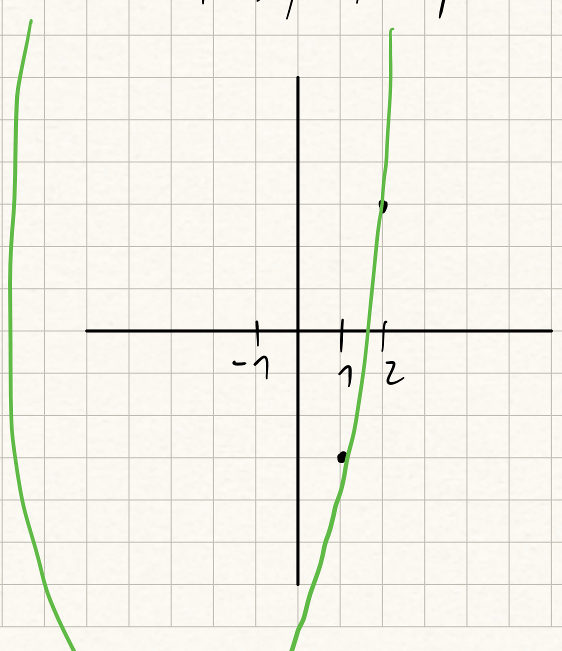
jsem v $\mathbb{Z}_5$, takže

$x^{20} = x^{16} = x^{12} = x^{8} = \underline{\underline{x^{4}}}$

Redukujeme stupeň!

(x_1, y_1) (x_2, y_2) (x_3, y_3)

(9) a) $(-1, -9)$, $(1, -3)$, $(2, 3)$



$p(x) = ?$

$$P_1 = \frac{(x-x_2)\,(x-x_3)}{(x_1-x_2)\cdot(x_1-x_3)} = \frac{x^2-3x+2}{6}$$

$$P_2 = \frac{(x-x_1)\cdot(x-x_3)}{(x_2-x_1)\,(x_2-x_3)} = -x^2+x+2$$

$$P_3 = \frac{(x-x_1)\cdot(x-x_2)}{(x_3-x_1)\cdot(x_3-x_2)} = \frac{x^2-1}{3}$$

$$p = -4\cdot p_1(x) - 3\cdot p_2(x) + 3\,p_3(x) = 2x^2-3x+5$$