

$$A \begin{pmatrix} 2 & 1 & 0 & 0 \\ 4 & 2 & 0 & 0 \\ 4 & 3 & 0 & 1 \\ 0 & 1 & 4 & 3 \end{pmatrix}, p_A(t) = \begin{vmatrix} 2-t & 1 & 0 & 0 \\ 4 & 2-t & 0 & 0 \\ 4 & 3 & -t & 1 \\ 0 & 1 & 4 & 3-t \end{vmatrix} = -t \begin{vmatrix} 2-t & 1 & 0 \\ 4 & 2-t & 0 \\ 0 & 1 & 3-t \end{vmatrix}$$

$$-t \cdot \begin{vmatrix} 2-t & 1 & 0 \\ 4 & 2-t & 0 \\ 4 & 3 & 1 \end{vmatrix} = -t \cdot \left((2-t) \cdot (3-t) - 4 \cdot (3-t) \right) - t \cdot \left((2-t)^2 - 4 \right) =$$

$$-t \cdot (2-t)^2 \cdot (3-t) + 12t - 4t^2 - t(2-t)^2 + 4t =$$

$$t^4 - 4t^3 + \cancel{1t^2} - 3t^3 + 12t^2 - \cancel{12t} + \cancel{12t} - \cancel{4t^2} - \cancel{16} + 16t - 4t^2 + \cancel{16} =$$

$$t^4 - 7t^3 + 8t^2 + 16t = t \cdot (t^3 + 5t^2 + 8t + 16) = 0 \quad t \cdot (t+1)^3 = 0$$

$$\lambda_1 = 0, \lambda_2 = (-1) \rightarrow 4$$

$$A_0 \begin{pmatrix} 2 & 1 & 0 & 0 \\ 4 & 2 & 0 & 0 \\ 4 & 3 & 0 & 1 \\ 0 & 1 & 4 & 3 \end{pmatrix} \sim \begin{pmatrix} 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 2 & 2 & 0 & 1 \\ 0 & 1 & 4 & 3 \end{pmatrix} \sim \begin{pmatrix} 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \begin{aligned} x_4 &= t = t \\ x_3 &= -\frac{t}{2} = 2t \\ x_2 &= -t = 4t \\ x_1 &= \frac{t}{2} = t \end{aligned}$$

$$v_1 = c \cdot (1, 4, 2, 1)^T$$

$$A_4 \begin{pmatrix} -2 & 1 & 0 & 0 \\ 4 & -2 & 0 & 0 \\ 4 & 3 & -4 & 1 \\ 0 & 1 & 4 & -1 \end{pmatrix} \sim \begin{pmatrix} 3 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 4 & 4 \end{pmatrix} \sim \begin{pmatrix} 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \begin{aligned} x_4 &= t = 1 \\ x_3 &= -t = 4 \\ x_2 &= 0 = 0 \\ x_1 &= 0 = 0 \end{aligned}$$

$$v_2 = c \cdot (0, 0, 4, 1)^T$$

Není diagonalizovatelná, jelikož λ_2 má alg. násobnost 3, zatímco geometrickou pouze 1.

V Z_m A¹⁰⁰⁰?

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 5 & 6 & 7 \end{pmatrix} \quad P_A(t) = \begin{vmatrix} 1-t & 2 & 3 \\ 3 & 4-t & 5 \\ 5 & 6 & 7-t \end{vmatrix} = -t^3 + 12t^2 - 35t + 28 + 50 + 54 - 60 + 15t - 30 + 30t - 42 + 6t$$

$$= 10t^3 + t^2 + t = 0$$

$$\frac{10 \pm \sqrt{1-40}}{20} = \frac{10 \pm \sqrt{5}}{20} = \frac{10 \pm 4}{9} = \begin{cases} \frac{3}{9} = 4 \\ \frac{6}{9} = 8 \end{cases}$$

$$t \cdot (10t^2 + t + 1) = 0$$

$$\lambda_1 = 0$$

$$\lambda_2 = 4$$

$$\lambda_3 = 8$$

→ Tři vlast. čísla pro matici 3x3

$$A_0 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 5 & 6 & 7 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 3 \\ 0 & 9 & 7 \\ 0 & 7 & 3 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 3 \\ 0 & 7 & 3 \\ 0 & 1 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 10 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

$$v_1 = c \cdot (1, 9, 1)^T$$

$$A_4 = \begin{pmatrix} 8 & 2 & 5 \\ 3 & 0 & 5 \\ 5 & 6 & 3 \end{pmatrix} \sim \begin{pmatrix} 3 & 0 & 5 \\ 3 & 7 & 0 \\ 5 & 6 & 3 \end{pmatrix} \sim \begin{pmatrix} 3 & 0 & 5 \\ 0 & 7 & 6 \\ 2 & 6 & 9 \end{pmatrix} \sim \begin{pmatrix} 1 & 5 & 7 \\ 0 & 7 & 6 \\ 0 & 7 & 6 \end{pmatrix} \sim \begin{pmatrix} 1 & 5 & 7 \\ 0 & 7 & 6 \\ 0 & 0 & 0 \end{pmatrix}$$

$$x_3 = t$$

$$7x_2 = -6t \quad x_2 = -\frac{6}{7}t = -4t = 7t$$

$$x_1 = -35t - 7t = 2t$$

$$v_2 = c \cdot (2, 7, 1)^T$$

$$A_8 = \begin{pmatrix} 4 & 2 & 3 \\ 3 & 7 & 5 \\ 5 & 6 & 10 \end{pmatrix} \sim \begin{pmatrix} 1 & 6 & 9 \\ 3 & 7 & 5 \\ 5 & 6 & 10 \end{pmatrix} \sim \begin{pmatrix} 1 & 6 & 9 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 6 & 9 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$x_3 = t$$

$$x_2 = 10t$$

$$x_1 = -69t = 8t$$

$$v_3 = c \cdot (8, 10, 1)^T$$

$$R = \begin{pmatrix} 1 & 2 & 8 \\ 9 & 7 & 10 \\ 1 & 1 & 1 \end{pmatrix} \rightsquigarrow R^{-1} = \begin{pmatrix} 2 & 7 & 2 \\ 3 & 1 & 10 \\ 6 & 3 & 0 \end{pmatrix}, \quad J = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 8 \end{pmatrix}$$

			0	0	0	2	7	2
			0	1	0	3	1	10
			0	0	1	6	3	0
1	2	8	0	2	8	10	4	9
4	7	10	0	7	10	4	4	4
1	1	1	0	1	1	9	4	10

$$A^{1000} = \begin{pmatrix} 10 & 4 & 9 \\ 4 & 4 & 4 \\ 9 & 4 & 10 \end{pmatrix}$$