

| Funkee                         | Derivee                            | Primitivní Funkee                         |
|--------------------------------|------------------------------------|-------------------------------------------|
| $f(x)$                         | $f'(x)$                            | $\int f(x)$                               |
| $x^n$                          | $n \cdot x^{n-1} \quad (n \neq 0)$ | $\frac{1}{n+1} x^{n+1} \quad (n \neq -1)$ |
| $\frac{1}{x} \quad (x \neq 0)$ | $-\frac{1}{x^2}$                   | $\ln x $                                  |
| $e^x$                          | $e^x$                              | $e^x$                                     |
| $\sin x$                       | $\cos x$                           | $-\cos x$                                 |
| $\cos x$                       | $-\sin x$                          | $\sin x$                                  |
| $\ln x$                        | $\frac{1}{x}$                      | $x \cdot \ln x - x$                       |
| $\operatorname{tg} x$          | $\frac{1}{\cos^2 x}$               |                                           |
| $\operatorname{arctg} x$       | $\frac{1}{1+x^2}$                  |                                           |

interval  
není roztahy →

### Společité integrál:

a)  $\int (x^2 + 2x) dx$

b)  $\int (x+2)^2 dx$

c)  $\int e^x - e^{-x} dx$

d)  $\int \sin x - \cos x dx$

e)  $\int \frac{1+x}{\sqrt{x}} dx$

f)  $\int \tan^2 x dx$

g)  $\int x \cdot e^{-x^2} dx$

$f(g(x)) = (f'(g(x)) \cdot g'(x)) dx$

a)  $\frac{x^3}{3} + x^2 + c$

b)  $\begin{aligned} & \rightarrow -x^2 + hx + h \rightarrow \frac{x^3}{3} + 2x^2 + hx + c \\ & t = x+2 \\ & dt = dx = \int t^2 dt = \frac{t^3}{3} = \frac{(x+2)^3}{3} + c \end{aligned}$

c)  $\int e^x dx + \int e^{-x} dx = e^x + e^{-x} + c$

d)  $\left( \int \sin x - \int \cos x \right) dx = -\cos x - \sin x + c$

e)  $\int \frac{1+x}{x^{\frac{1}{2}}} dx = \int \left( x^{-\frac{1}{2}} + x^{\frac{1}{2}} \right) dx = \int x^{-\frac{1}{2}} dx + \int x^{\frac{1}{2}} dx = 2 \cdot x^{\frac{1}{2}} + \frac{2}{3} x^{\frac{3}{2}} + c$

f)  $\int \frac{\sin^2 x}{\cos^2 x} dx = \int \frac{1-\cos^2 x}{\cos^2 x} dx = \int \frac{1}{\cos^2 x} - 1 dx = \operatorname{tg} x - x + c$

g)  $-\frac{1}{2} \int e^{-x^2} \cdot (-2x) dx = -\frac{1}{2} \cdot e^{-x^2} + c$

$\begin{aligned} t &= -x^2 \\ dt &= -2x dx \\ &= \int e^t \cdot \frac{dt}{-2} = -\frac{1}{2} \int e^t dt = -\frac{1}{2} e^t + c = -\frac{1}{2} e^{-x^2} + c \end{aligned}$

$\int (x+2)^2 dx$   
+  $\int t^2 dt$

$f(g(x)) = (f'(g(x)) \cdot g'(x)) dx$

## Spočítejte integrál:

a)  $\int \tan x \, dx$

b)  $\int \sin^2 x \, dx$

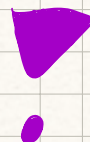
c)  $\int \cos^2 x \, dx$

$$\cos x = t$$

a)  $\int \frac{\sin x}{\cos x} \, dx = - \int \frac{1}{\cos x} \cdot (-\sin x) \cdot dx = - \ln |\cos x| + C$



$$t = \cos x \\ dt = -\sin x \, dx$$



$$- \int \frac{-\sin x}{\cos x} \, dx = - \int \frac{1}{t} \, dt = - \ln |t| = - \ln |\cos x| + C$$

b)  $\left. \begin{array}{l} \sin^2 x + \cos^2 x = 1 \\ \cos^2 x - \sin^2 x = \cos 2x \end{array} \right\} \sin^2 x = \frac{1}{2} (1 - \cos 2x)$

$$\frac{1}{2} \int 1 - \cos 2x \, dx = \frac{1}{2} (x - \sin x \cos x) + C$$

$$\int \cos 2x \, dx = \frac{1}{2} \sin 2x = \sin x \cos x \\ t = 2x \\ dt = 2 \, dx$$

$$f(x) \cdot g(x) = \int f(x) \cdot g'(x) \, dx + \int f'(x) g(x) \, dx$$

## Derivace plus partes:

$$\int f(x) \cdot g'(x) \, dx = f(x) \cdot g(x) - \int f'(x) g(x) \, dx$$

## Spočítejte integrál:

a)  $\int x \cdot e^x \, dx$

b)  $\int x \cdot \sin x \, dx$

$$\begin{array}{ll} f & g \\ \downarrow & \downarrow \\ f = x & g' = e^x \\ f' = 1 & g = e^x \end{array}$$

a)  $\int x \cdot e^x \, dx = x \cdot e^x - \int 1 \cdot e^x \, dx = x \cdot e^x - e^x + C$

$$\begin{array}{ll} f & g \\ \downarrow & \downarrow \\ f = x & g' = \sin x \\ f' = 1 & g = -\cos x \end{array}$$

b)  $\int x \cdot \sin x \, dx = -x \cdot \cos x - \int 1 \cdot (-\cos x) \, dx = -x \cos x + \sin x + C$

## Opakování: Spočítejte integrál:

$$\int x \cdot e^{-x^2} \, dx =$$

$$t = -x^2 \\ dt = -2x \, dx$$

$$\int \ln x \, dx = \int \ln x \cdot 1 \, dx$$

$$\begin{array}{ll} f & g' \\ \downarrow & \downarrow \\ f = \ln x & f' = \frac{1}{x} \\ g = x & g' = 1 \end{array}$$