$(\text{konst})' = 0$

$(x)' = 1$

$(x^n)' = n \cdot x^{n-1}$

$(e^x)' = e^x$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\ln x)' = \frac{1}{x}$

$f: \mathbb{R} \to \mathbb{R}$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Aritmetika:

$(\alpha f)' = \alpha \cdot f'$

$(f+g)' = f' + g'$

$(f \cdot g)' = f' \cdot g + f \cdot g'$

$\left(\frac{f}{g}\right)' = \frac{f' \cdot g - f \cdot g'}{g^2}$

Složená funkce:

$f(g(a))' = f'(g(a)) \cdot g'(a)$

$$\left(\frac{1}{x}(a)\right)' = \lim_{h \to 0} \frac{\frac{1}{a+h} - \frac{1}{a}}{h} = \frac{\frac{a-(a+h)}{a \cdot (a+h)}}{h} = \frac{\frac{-h}{a^2+ah}}{\frac{h}{1}} = \frac{-1}{a^2+ah} = -\frac{1}{a^2}$$

$\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$

$(\sqrt{x})' = (x^{\frac{1}{2}})' = \frac{1}{2\sqrt{x}} = \frac{1}{2} \cdot x^{-\frac{1}{2}}$

protože složená funkce

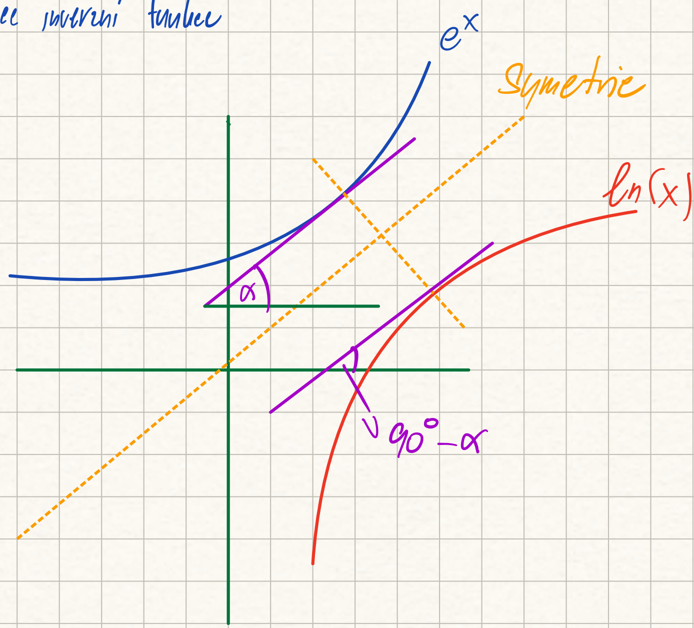
$$\left(\frac{f}{g}\right)' = \left(f \cdot \frac{1}{g}\right)' = f' \cdot \frac{1}{g} + f \cdot \left(\frac{1}{g}\right)' = f' \cdot \frac{1}{g} + f \cdot \frac{-1}{g^2} \cdot g' = \frac{f' \cdot g - f \cdot g'}{g^2}$$

Věta:

$f(x) = y \qquad (f^{-1}(y))' = \frac{1}{f'(x)}$ Derivace inverzní funkce

$e^x = y$

$(\ln y)' = \frac{1}{e^x} = \frac{1}{y}$



2derivujte:

$\sin^2 x' + \cos^2 x' = (\sin x \cdot \sin x)' + (\cos x \cdot \cos x)' = \sin x \cdot \cos x + \cos x \cdot \sin x +$

$- \cos x \cdot \sin x - \sin x \cdot \cos x =$

nebo //

a) $(\sin^2 x + \cos^2 x)' = (1)' = 0$

b) $(x^x)' = (e^{x \cdot \ln x})' = \underbrace{e^{x \ln x}}_{x^x} \cdot (x \cdot \ln x)' = x^x \cdot (\ln x + x \cdot \frac{1}{x}) = x^x \cdot (1 + \ln x)$

! c) $(x^{\ln x})' = x^{\ln x} \cdot \frac{2 \ln x}{x}$

! d) $((\ln x)^x)' = (\ln x)^x \cdot (\ln \cdot \ln x) + (\ln x)^{x-1}$

e) $(\arctan \frac{1}{x})'$ $\longrightarrow$ $(\tan x)' = \left(\frac{\sin x}{\cos x}\right)' = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = 1 + \tan^2 x$

$\left(\frac{\sin x}{\cos x}\right)^2 = (\tan x)^2$

$\to (\arctan x)' \to \tan x = y \Rightarrow (\arctan y)' = \frac{1}{1 + \tan^2 x} = \frac{1}{1+y^2}$

$\to (\arctan \frac{1}{x})' = \frac{1}{1 + (\frac{1}{x})^2} \cdot \left(-\frac{1}{x^2}\right) = \frac{-1}{\frac{x^2+1}{x^2} \cdot x^2} = -\frac{1}{1+x^2}$

2derivujte:

$f(g(x))' = f'(g(x)) \cdot g'$

a) $(\cos^3(2x))' = $ sl. fun. $3 \cdot \cos^2 2x \cdot (-\sin 2x) \cdot 2 = -6 \cdot \sin 2x \cdot \cos^2 2x$

$\nearrow g'$

b) $(\sin \sqrt{x-1})' = \cos \sqrt{x-1} \cdot \frac{1}{2\sqrt{x-1}} \cdot 1$

c) $\left(\sqrt{\frac{1-x^2}{1+x^2}}\right)'$

d) $(x^2 \cdot 2^x)' = 2x \cdot 2^x + x^2 \cdot (2^x)' = 2x \cdot 2^x + x^2 \cdot 2^x \cdot \ln 2$

$(c^x)' = (e^{x \ln c})' = \underbrace{e^{x \ln c}}_{c^x} \cdot \ln c$

e) $\left(x^{x^x}\right)'$

$\underbrace{e^{x \ln x}} = \underbrace{a} = x^x$

$\ln e^{x \ln x} = \ln_e (a)$

$x \ln x \cdot \cancel{\ln e} = \ln e^{(a)}$

$\ln (x^x) = \ln_e (a)$

$x^x = a$

$\boxed{e^{a \ln b} = x} = b^a$

$\ln_e e^{a \ln b} = \ln x$

$a \ln b \; \cancel{\ln e} = \ln x$

$\ln b^a = \ln x$

$b^a = x$