

Spätere Limits:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \dots$$

$$\sin x = \sum_{i=0}^{\infty} \frac{(-1)^i x^{2i+1}}{(2i+1)!}$$

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = \lim_{x \rightarrow 0} \frac{1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \dots - 1}{x} = 1 + \frac{x}{2} + \frac{x^2}{6} + \dots = 1 + \frac{0}{2} + \frac{0}{6} + \dots = \underline{\underline{1}}$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = \frac{x - \frac{x^3}{6} + \frac{x^5}{120}}{x} = 1 - \frac{x^2}{6} + \frac{x^4}{120} = 1 - 0 + 0 \dots = \underline{\underline{1}}$$

$$\sin x = \sum_{i=0}^{\infty} \frac{(-1)^i x^{2i+1}}{(2i+1)!}$$

$$\frac{\sin x}{\cos x} \cdot \frac{x}{1} =$$

$$\lim_{x \rightarrow 0} \frac{\tan x}{x} = \frac{\sin x}{\cos x} \cdot \frac{1}{x} = \frac{\sin x}{x \cos x} = \lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \lim_{x \rightarrow 0} \frac{1}{\cos x} = 1 \cdot 1 = \underline{\underline{1}}$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{0}{0} \stackrel{LH}{=} \lim_{x \rightarrow 0} \frac{(1 - \cos x)'}{(x^2)'} = \frac{\sin x}{2x} = \frac{1}{2} \cdot \frac{\sin x}{x} = \frac{1}{2} \cdot 1 = \underline{\underline{\frac{1}{2}}}$$

$$\lim_{x \rightarrow 0} \frac{\ln(x+1)}{x} = \begin{cases} g(x) = \ln(x+1) \\ \lim_{x \rightarrow 0} \ln(x+1) = 0 \end{cases} \rightarrow \lim_{x \rightarrow 0} \frac{x}{x} \stackrel{LH}{=} \lim_{x \rightarrow 0} \frac{1}{1} = \underline{\underline{1}}$$

$$\lim_{x \rightarrow +\infty} \frac{2x^5 + 7x + 1}{3x^5 + 4x^3 + x^2} = \frac{x^5 \cdot (2 + \frac{7}{x^4} + \frac{1}{x^5})}{x^5 \cdot (3 + \frac{4}{x^2} + \frac{1}{x^3})} = \underline{\underline{\frac{2}{3}}}$$

$$(x^2 - x - 2) \cdot (x^2 - x - 2) = x^4 - x^3 - 2x^2 - x^3 + x^2 + 2x - 2x^2 + 2x + 4 = x^4 - 2x^3 - 3x^2 + 4x + 4$$

$$\lim_{x \rightarrow 2} \frac{(x^2 - x - 2)^2}{x^3 - 12x + 16} = \frac{(x-2)^2 \cdot (x+1)^2}{(x-2) \cdot (x^2 + 2x - 8)} = \frac{(x-2)^2 \cdot (x+1)^2}{(x-2)^2 \cdot (x+4)} = \frac{(x+1)^2}{x+4} = \frac{6}{4} = \underline{\underline{\frac{3}{2}}}$$

$$\begin{aligned} x^3 - 12x + 16 &= x^3 - 2x^2 + 2x^2 - 12x + 16 \\ &= 0 + 2x^2 + 2x^2 - 12x + 16 \\ &= 0 + 4x^2 - 8x + 16 \\ &= 0 + 4x^2 - 8x + 16 \end{aligned}$$

$$\lim_{x \rightarrow 3} \frac{x^2 - 5x + 6}{x^2 - 8x + 15} = \frac{(x-2) \cdot \cancel{(x-3)}}{(x-5) \cdot \cancel{(x-3)}} = \frac{x-2}{x-5} = \frac{1}{-2} = \underline{\underline{-\frac{1}{2}}}$$

$$\begin{aligned} &= (1+x+2x+2x^2) \cdot (1+3x) = (2x^2+3x+1) \cdot (1+3x) \\ &= 2x^2+3x+1 + 6x^3+9x^2+3x = 6x^3+11x^2+6x+1 \end{aligned}$$

$$\lim_{x \rightarrow 0} \frac{(1+x) \cdot (1+2x) \cdot (1+3x) - 1}{x} = \frac{6x^3 + 11x^2 + 6x + 1 - 1}{x} = 6x^2 + 11x + 6 \Rightarrow \underline{\underline{6}}$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x} = \frac{1+x-1}{x \cdot \sqrt{1+x} + 1} = \frac{x}{x \sqrt{1+x} + 1} = \frac{1}{\sqrt{1+x} + 1} = \underline{\underline{\frac{1}{2}}}$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt[3]{1+x} - \sqrt[3]{1-x}} = \frac{\frac{(1+x) - (1-x)}{\sqrt{1+x} + \sqrt{1-x}}}{\frac{(1+x) - (1-x)}{\sqrt[3]{(1+x)^2 + \sqrt[3]{(1+x) \cdot (1-x)} + \sqrt[3]{(1-x)^2}}} = \frac{(1+x) - (1-x)}{\sqrt{1+x} + \sqrt{1-x}} = \frac{2x}{2\sqrt{1+x}} = \frac{1}{2}$$

$$(A-B) \cdot (A^2 + AB + B^2) = A^3 - B^3$$

$$(A-B) \cdot (A+B) = A^2 - B^2$$

$$\lim_{x \rightarrow \infty} \sqrt{x + \sqrt{x + \sqrt{x}}} - \sqrt{x} = \frac{x + \sqrt{x + \sqrt{x}} - x}{\sqrt{x + \sqrt{x + \sqrt{x}}} + \sqrt{x}} \cdot \frac{\frac{1}{\sqrt{x}}}{\frac{1}{\sqrt{x}}} = \frac{\sqrt{\frac{x + \sqrt{x}}{x}}}{\sqrt{\frac{x + \sqrt{x + \sqrt{x}}}{x}} + 1} = \frac{\sqrt{1 + \frac{\sqrt{x}}{x}}}{\sqrt{1 + \frac{1}{\sqrt{x}}} + 1} = \frac{1}{2}$$

$$\lim_{n \rightarrow \infty} \ln\left(1 + \frac{1}{\sqrt{n}}\right) = \ln\left(\frac{\sqrt{n} + 1}{\sqrt{n}}\right) = \ln\left(\frac{n + \sqrt{n}}{n}\right)$$

Nemůžeme použít univerzální limitu, protože limitní funkce

$$\lim_{x \rightarrow \infty} \ln\left(1 + \frac{1}{\sqrt{x}}\right)$$

$$f(x) = \ln(x)$$

$$g(x) = 1 + \frac{1}{\sqrt{x}}$$

$$\lim_{x \rightarrow \infty} 1 + \frac{1}{\sqrt{x}} = 1$$

$$g: P(A, \delta_1), f: U(\delta_2)$$

$$\lim_{x \rightarrow 1} \ln(x) = 0$$

Limitní složené funkce lze použít, jestliže splňují podmínky:

1) f je spojitá v limitním bodě

2) $\exists \delta: 0 < \delta < \delta_1$ t.j. $U(g(P(A, \delta)))$

Spočítejte limity:

$$\lim_{x \rightarrow \infty} \sqrt{1 + \frac{1}{x^2}} = 1$$

$$a^x = e^{\log a^x} = e^{x \log a}$$

$$\lim_{x \rightarrow 0} \frac{\ln(\sin x + 1)}{x}$$

$$f(x) = \frac{\ln(x+1)}{x}$$

$$g(x) = \sin x$$

$$\lim_{x \rightarrow 0} \sin x = 0$$

$$\lim_{x \rightarrow 0} \frac{\ln(x+1)}{x} = 1$$

$$\lim_{x \rightarrow 0} (x+1)^{\frac{1}{x}} = e^{\frac{1}{x} \log(x+1)}$$

$$\lim_{x \rightarrow 0} \frac{\log(x+1)}{x} = 1$$

$$\lim_{x \rightarrow 1} e^x - e^1 = e$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{n^2 + 1} = \sqrt[n]{n^2 \cdot \left(1 + \frac{1}{n^2}\right)} = n^{\frac{2}{n}} \cdot \sqrt[n]{1 + \frac{1}{n^2}} = n^0 \cdot \sqrt[n]{1 + 0} = 1 \cdot 1 = 1$$

$$\begin{aligned} \lim_{n \rightarrow \infty} \sqrt{n} \cdot (\sqrt{n+1} - \sqrt{n-1}) &= \sqrt{n^2+n} - \sqrt{n^2-n} \cdot \frac{\sqrt{n^2+n} + \sqrt{n^2-n}}{\sqrt{n^2+n} + \sqrt{n^2-n}} = \frac{n^2+n - n^2+n}{\sqrt{n^2+n} + \sqrt{n^2-n}} = \frac{2n}{\sqrt{n^2+n} + \sqrt{n^2-n}} = \\ &= \frac{2n}{\sqrt{n^2 \cdot \left(1 + \frac{1}{n}\right)} + \sqrt{n^2 \cdot \left(1 - \frac{1}{n}\right)}} = \frac{2n}{n \cdot \sqrt{1 + \frac{1}{n}} + n \cdot \sqrt{1 - \frac{1}{n}}} = \frac{2n}{2n} = 1 \end{aligned}$$

$$(A-B) \cdot (A^2 + AB + B^2) = A^3 - B^3$$

$$\lim_{x \rightarrow 0} \frac{\sqrt[3]{1+x} - 1}{x} \cdot \frac{(1+x)^{\frac{2}{3}} + 1 + (1+x)^{\frac{4}{3}}}{(1+x)^{\frac{2}{3}} + 1 + (1+x)^{\frac{4}{3}}} = \frac{x}{x \cdot ((1+x)^{\frac{2}{3}} + 1 + (1+x)^{\frac{4}{3}})} = \frac{1}{(1+x)^{\frac{2}{3}} + 1 + (1+x)^{\frac{4}{3}}} = \frac{1}{3}$$

$\lim_{x \rightarrow 0} \frac{1}{x^n}$ $n \in \mathbb{N}$ $\begin{cases} n \text{ liché} \\ n \text{ sudé} \end{cases}$ $\rightarrow$ levé okolí 0 a pravé okolí 0 nemají stejný limit, neexistuje
 $a^x = e^{\log a^x} = e^{x \log a}$
 $\rightarrow$ levé i pravé okolí je kladné, tedy má stejný limit $+\infty$

$\lim_{x \rightarrow 0} \frac{1}{x}$ $\begin{cases} \lim_{x \rightarrow 0^+} \frac{1}{x} = +\infty \\ \lim_{x \rightarrow 0^-} \frac{1}{x} = -\infty \end{cases}$ $\rightarrow$ Limita tak neexistuje

Tabulka derivací:

$$(\text{const})' = 0$$

$$x' = 1$$

$$x^n = n \cdot x^{n-1}$$

$$(e^x)' = e^x$$

$$(\sin x)' = \cos x$$

$$(\cos x)' = -\sin x$$

$$\log x = \frac{1}{x}$$

$$\text{Derivace: } (\alpha f)' = \alpha f'$$

$$(f+g)' = f' + g'$$

$$(f \cdot g)' = f'g + f \cdot g'$$

$$(f/g)' = \frac{f'g - f \cdot g'}{g^2}$$

$$f(g)' = f'(g) \cdot g'$$

$$f \cdot g = \int f' \cdot g + \int f \cdot g'$$

Zderivujte: $\rightarrow$ nebo: $(\sin^2 x)' + (\cos^2 x)' = (\sin x \cdot \sin x)' + (\cos x \cdot \cos x)' = \sin x \cdot \cos x + \cos x \cdot \sin x - \cos x \cdot \sin x - \sin x \cdot \cos x = 0$

$$(\sin^2 x + \cos^2 x)' = (\sin^2 x + 1 - \sin^2 x)' = 0$$

$$(x^x)' = e^{x \log x} = e^{x \log x} \cdot (x \log x)' = x^x \cdot (\log x + x \cdot \frac{1}{x}) = x^x \cdot \log x + x^x = x^x \cdot (\log x + 1)$$

$$(x^{\log x})' = (e^{\log x \cdot \log x})' = (e^{\log x \cdot \log x})' = (e^{\log^2 x})' = x^{\log x} \cdot \frac{2 \log x}{x} \quad f(x) = e^x \quad g(x) = \log x \cdot \log x$$

$$f'(x) = \frac{\log x}{x} + \frac{\log x}{x}$$

$$g'(x) = \frac{2 \log x}{x}$$

$$(\ln(x)^x)' = ?$$

$$\begin{aligned}
 \left(\sqrt{\frac{(x+1)^2}{x^2+1}} \right)' &= \left(\left(\frac{(x+1)^2}{x^2+1} \right)^{\frac{1}{2}} \right)' = \frac{1}{2} \cdot \left(\left(\frac{(x+1)^2}{x^2+1} \right)^{-\frac{1}{2}} \right) \cdot \frac{((x+1)^2)' \cdot (x^2+1) - (x+1)^2 \cdot (x^2+1)'}{(x^2+1)^2} = \\
 &= \frac{1}{2} \cdot \frac{|x+1|^{-1}}{(x^2+1)^{-\frac{1}{2}}} \cdot \frac{\cancel{2} \cdot (x+1) \cdot 1 \cdot (x^2+1) - (x+1)^2 \cdot \cancel{2}x}{(x^2+1)^2} = \frac{|x+1|^{-1}}{(x^2+1)^{-\frac{1}{2}}} \cdot \frac{(x+1) \cdot (x^2+1) - (x+1)^2 \cdot x}{(x^2+1)^2} = \\
 &= \frac{|x+1|^{-1}}{(x^2+1)^{-\frac{1}{2}}} \cdot \frac{1-x^2}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^{\frac{3}{2}} \cdot |x+1|}
 \end{aligned}$$

$x^2+2x+1 = x^2+2x^2+x$
 $\cancel{x^2} + x^2 + \cancel{x} + 1$

$$\ln(\sin(e^x))' = \frac{1}{\sin(e^x)} \cdot \sin(e^x)' = \frac{1}{\sin(e^x)} \cdot \cos(e^x) \cdot e^x = \frac{\cos(e^x)}{\sin(e^x)} \cdot e^x = \cot(e^x) \cdot e^x$$

$$(x \cos x + \sin(2x^2))' = x' \cdot \cos x + x \cdot (\cos x)' + \cos 2x^2 \cdot (2x^2)' = \cos x - x \cdot \sin x + \cos 2x^2 \cdot 4x$$

Vnější x^3
vnitřní $\cos 2x$

$$(\cos^3(2x))' = 3 \cos^2 2x \cdot (\cos 2x)' = 3 \cos^2 2x \cdot (-\sin 2x) \cdot 2 = -6 \cos^2 2x \cdot \sin 2x.$$

Musím chodit postupně zevnitř dovnitř. Tedy nejdřív derivuji

a^3 , pak $\cos 2x$, pak $2x$... Poslední poznámka věta o derivaci složené funkce.

$$(\sin \sqrt{x-1})' =$$

$f(x) = \sin x$ $g(x) = (x-1)^{\frac{1}{2}}$
 $f'(x) = \cos x$ $g'(x) = \frac{1}{2\sqrt{x-1}}$

$$\cos \sqrt{x-1} \cdot \frac{1}{2\sqrt{x-1}} = \frac{\cos \sqrt{x-1}}{2\sqrt{x-1}}$$

Pauzuj substituci a pak derivuj! Sazba

$$\left(\sqrt{\frac{1-x^2}{1+x^2}} \right)' =$$

$\frac{1-x^2}{1+x^2} = d$ $(\sqrt{d})' = (d^{\frac{1}{2}})' = \frac{1}{2\sqrt{d}} \cdot d'$
 $\rightarrow$ je spojitá

$$= \frac{1}{2\sqrt{\frac{1-x^2}{1+x^2}}} \cdot \left(\frac{1-x^2}{1+x^2} \right)' = \frac{1}{2\sqrt{\frac{1-x^2}{1+x^2}}} \cdot \frac{-2x \cdot (1+x^2) - ((1-x^2) \cdot 2x)}{(1+x^2)^2} =$$

$$= \frac{-4x}{2\sqrt{\frac{1-x^2}{1+x^2}} \cdot (1+x^2)^2} = - \frac{2x}{\sqrt{\frac{1-x^2}{1+x^2}} \cdot (1+x^2)^2}$$

$e^{\log 2^x} = (e^{\log 2^x})' = e^{\log 2^x} \cdot (x \log 2)' = e^{\log 2^x} \cdot \log 2$
 $\log(e) \cdot (x)' = \log 2$

$$(x^2 \cdot 2^x)' = 2x \cdot 2^x + x^2 \cdot (2^x)' = 2x \cdot 2^x + x^2 \cdot 2^x \cdot \log 2$$

Spočítejte limitu:

$$\lim_{x \rightarrow 1} \frac{x^3 - x^2 - x + 1}{x^3 + x^2 - x - 1} = \frac{0}{0} \xrightarrow{L'H} \lim_{x \rightarrow 1} \frac{(x^3 - x^2 - x + 1)'}{(x^3 + x^2 - x - 1)'} = \frac{3x^2 - 2x - 1}{3x^2 + 2x - 1} = \frac{0}{5} = 0$$

$$\lim_{x \rightarrow 1} \frac{x^3 + x^2 - x - 1}{x^3 + 2x^2 - x - 2} = \frac{0}{0} \xrightarrow{L'H} \lim_{x \rightarrow 1} \frac{(x^3 + x^2 - x - 1)'}{(x^3 + 2x^2 - x - 2)'} = \frac{3x^2 + 2x - 1}{3x^2 + 4x - 1} = \frac{4}{6} = \frac{2}{3}$$

$$\lim_{x \rightarrow 0} \frac{\sqrt[3]{1+x} - 1}{x} = \frac{0}{0} \xrightarrow{L'H} \lim_{x \rightarrow 0} \frac{\left((1+x)^{\frac{1}{3}} - 1 \right)'}{(x)'} = \frac{\frac{1}{3} \cdot (1+x)^{-\frac{2}{3}} \cdot (1+x)'}{1} = \frac{1+x}{3} = \frac{1}{3}$$

$\rightarrow$ uvažuj! jsi se vzepřel pomocí $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = \frac{0}{0} \xrightarrow{L'H} \lim_{x \rightarrow 0} \frac{(\sin x)'}{x'} = \frac{\cos x}{1} = 1$$

$$\lim_{x \rightarrow 0} x \cdot \sin \frac{1}{x} = \frac{\sin \frac{1}{x}}{\frac{1}{x}} = \frac{0}{0} \xrightarrow{\text{L'H}} \lim_{x \rightarrow 0} \frac{(\sin \frac{1}{x})'}{(\frac{1}{x})'} = \frac{\cos \frac{1}{x} \cdot (-x^{-2})}{-x^{-2}} = \cos \frac{1}{x} = 1$$

cos je spojita (volse)

$$\lim_{x \rightarrow \infty} \frac{\ln(1 + \sqrt{x})}{\ln(1 + \sqrt[3]{x})} = \frac{\cancel{\ln 1} \cdot \ln \sqrt{x}}{\cancel{\ln 1} \cdot \ln \sqrt[3]{x}} = \frac{\frac{1}{2} \cancel{\ln x}}{\frac{1}{3} \cancel{\ln x}} = \frac{\frac{1}{2}}{\frac{1}{3}} = \frac{3}{2}$$

Vyšetřete průběh funkce:

$$x^3 - 12x + 16$$

$$D(f): \mathbb{R}$$

Průběh os:

$$y: [0, 16]$$

$$x: x^3 - 12x + 16 = 0$$

$$(x-2) \cdot (x^2 + 2x - 8) = 0$$

$$(x-2) \cdot (x-2) \cdot (x+4) = 0$$

$$\begin{cases} [2, 0] \\ [-4, 0] \end{cases}$$

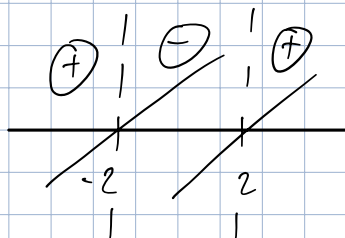
$$\begin{array}{r} x^3 - 12x + 16 = x^2 + 2x - 8 \\ -x^2 + 2x^2 \\ 2x^2 - 12x \\ -6x + 4x \\ 0 \quad -8x - 16 \\ +8x + 16 \end{array}$$

$$x_1 = 2, x_2 = -2$$

$$f' = 3x^2 - 12 = 3 \cdot (x^2 - 4) = 3 \cdot (x-2) \cdot (x+2)$$

$$f'' = 6x$$

- kde je to kladný, tak
je konvexní, kde záporný,
tak konkávní



- kde to je kladný, tak
funkce roste, kde záporný, tak klesá.

Vysvětlila funkci:

$$f(x) = \frac{x^2-1}{x^3-1} : D(f) : \mathbb{R} \setminus \{1\}$$

Průběhy vs:

$$y : x=0$$

$$\frac{0-1}{0-1} = 1 \quad [0,1]$$

$$x : y=0 : \frac{x^2-1}{x^3-1} = 0 \quad \begin{cases} [-1,0] \\ [1,0] \end{cases}$$

$$(x-1) \cdot (x+1) = 0$$

$$x = \pm 1$$

$$(x^3-1) \cdot (x^3-1) = x^6 - x^2 - x^3 + 1$$

$$x^6 - 2x^3 + 1$$

$$\lim_{x \rightarrow \infty} \frac{x^2-1}{x^3-1} = \lim_{x \rightarrow \infty} \frac{x^3 \cdot \left(\frac{1}{x} - \frac{1}{x^3}\right)}{x^3 \cdot \left(1 - \frac{1}{x^3}\right)} = \frac{\frac{1}{x} - \frac{1}{x^3}}{1 - \frac{1}{x^3}} = 0$$

$$\lim_{x \rightarrow -\infty} = \dots = 0$$

$$\lim_{x \rightarrow 1} \frac{x^2-1}{x^3-1} \stackrel{L'H}{=} \lim_{x \rightarrow 1} \frac{2x}{3x^2} = \frac{2}{3}$$

$$f'(x) = \frac{2x \cdot (x^3-1) - (x^2-1) \cdot 3x^2}{(x^3-1)^2} = \frac{2x^4 - 2x - 3x^4 + 3x^2}{(x^3-1)^2}$$

$$f'(x) = \frac{-x^4 + 3x^2 - 2x}{(x-1)^2 \cdot (x^2+x+1)^2} = \frac{(x-1) \cdot (-x^3 - x^2 + 2x)}{(x-1)^2 \cdot (x^2+x+1)^2} = \frac{(x-1) \cdot (-x^2 - 2x)}{(x-1)^2 \cdot (x^2+x+1)^2}$$

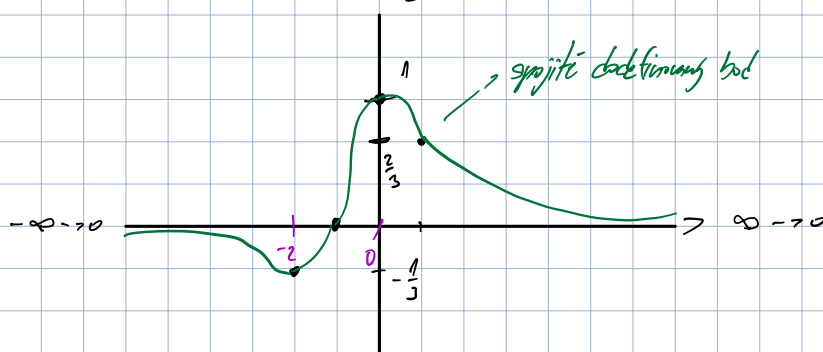
$$\frac{-x \cdot (x+2)}{(x^2+x+1)^2}$$

$$f'(x) = 0 \text{ pro } x = -2$$

$$x = 0$$

$$\ominus \quad | \quad \oplus \quad | \quad \ominus$$

$$-2 \quad 0$$



Spočítejte integrál:

$$\int x^2 + 2x \, dx = \frac{x^3}{3} + x^2 + c$$

$$\int (x+2)^2 \, dx = \int_{t=x+2}^{t=dx} t^2 \, dt = \frac{t^3}{3} = \frac{(x+2)^3}{3} + c$$

$$\int e^x - e^{-x} \, dx = \int e^x \, dx - \int e^{-x} \, dx = e^x - \int e^t - dx = e^x + \int e^t = e^x + e^{-x} + c$$

$$\int \sin x - \cos x \, dx = \int \sin x \, dx - \int \cos x \, dx = -\cos x - \sin x + c$$

$$\int \frac{1+x}{\sqrt{x}} dx = \int x^{-\frac{1}{2}} + x^{\frac{1}{2}} = \int x^{-\frac{1}{2}} dx + \int x^{\frac{1}{2}} dx = 2 \cdot x^{\frac{1}{2}} + \frac{2}{3} x^{\frac{3}{2}} + c$$

$$\int \tan^2 x dx = \int \frac{\sin^2 x}{\cos^2 x} dx = \int \frac{1 - \cos^2 x}{\cos^2 x} dx = \int \frac{1}{\cos^2 x} - \int 1 dx = \frac{1}{\tan x} - x + c$$

$$\int x \cdot e^{-x^2} = \begin{matrix} t = -x^2 \\ dt = -2x dx \end{matrix} = \int \underbrace{e^t}_{e^{-x^2}} \cdot \underbrace{dt}_{(-2x dx)} \cdot \left(-\frac{1}{2}\right) = -\frac{1}{2} e^t = -\frac{1}{2} e^{-x^2} + c$$

$e^{-x^2} \cdot (-2x dx) \cdot \left(-\frac{1}{2}\right)$
 $e^{-x^2} x dx \cdot (-2) \cdot \left(-\frac{1}{2}\right)$
1

$$\int \frac{1}{\cos x} dx = \int \frac{\sin x}{\cos x} dx \quad \begin{matrix} t = \cos x \\ dt = -\sin x \end{matrix} = \int \frac{1}{t} \cdot -1 \cdot dt = -\int \frac{1}{t} dt = -\ln|\cos x| + c$$

$$f(x) \cdot g(x) = \int f(x) \cdot g'(x) dx + \int f'(x) \cdot g(x) dx$$

$$\int x \cdot e^x dx = \begin{matrix} f(x) = x & g'(x) = e^x \\ f'(x) = 1 & g(x) = e^x \end{matrix} = x \cdot e^x - \int 1 \cdot e^x dx = (x-1) \cdot e^x + c$$

$$\int x \cdot \sin x dx = \begin{matrix} f(x) = x & g'(x) = \sin x \\ f'(x) = 1 & g(x) = -\cos x \end{matrix} = -\cos x \cdot x - \int 1 \cdot (-\cos x) = -\cos x \cdot x + \sin x + c$$

$$\int \frac{x}{x^2+1} dx \quad \begin{matrix} t = x^2+1 \\ dt = 2x dx \end{matrix} = \frac{1}{2} \int \frac{1}{t} dt = \frac{1}{2} \ln|t| = \frac{1}{2} \ln(x^2+1) + c$$

$$\int \frac{x^2+1}{x^2-1} dx = \int \frac{x^2-1+2}{x^2-1} dx = \int \frac{x^2-1}{x^2-1} + \frac{2}{x^2-1} dx = \int 1 + \frac{2}{x^2-1} dx =$$

$$\frac{2}{x^2-1} = \frac{A}{(x-1)} + \frac{B}{(x+1)} = \frac{A(x+1) + B(x-1)}{x^2-1}$$

$$A - B = 2 \quad A = 2, B = 0$$

$$A + B = 0$$

$$2x + Bx + B = 0 \quad x$$

$$2x + Bx = 0 \quad x$$

$$B = -2$$

$$A = 1$$

$$= \int 1 + \frac{1}{x-1} - \frac{1}{x+1} dx = x + \ln|x-1| + \ln|x+1| + c$$

$$\int \frac{x-2}{(x-1)^2} dx = \frac{x+2}{(x-1)^2} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} = \frac{A(x-1) + B}{(x-1)^2}$$

$B - A = -2$
 $A = 1$
 $B = -1$

$$\int \frac{1}{(x-1)} - \frac{1}{(x-1)^2} = \int \frac{1}{x-1} dx - \int \frac{1}{(x-1)^2} dx =$$

$$\begin{matrix} t = x-1 \\ dt = dx \end{matrix}$$

$$\ln|x| - \int t^{-2} dt = \ln|x| + \frac{1}{t} = \ln|x| + \frac{1}{x-1} + c$$

substitue o obtido em ambas, porque
(t-1)' = -t^2, já que t = x-1

$$\int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx \quad \begin{array}{l} t = e^x + e^{-x} \\ dt = e^x - e^{-x} dx \end{array} = \int \frac{1}{t} dt = \ln|t| = \ln|e^x + e^{-x}|$$

$$\int \ln^2 x \, dx = \int \overset{f}{\ln^2 x} \cdot \overset{g'}{1} dx \quad \begin{array}{l} f = \ln^2 x \\ f' = 2 \cdot \ln x \cdot \frac{1}{x} \end{array} \quad \begin{array}{l} g' = 1 \\ g = x \end{array} = \ln^2 x \cdot x - \int \frac{2 \cdot \ln x}{x} \cdot x = \ln^2 x \cdot x - 2 \int \ln x =$$

$$\ln^2 x \cdot x - 2x(\ln x - 1) = x \cdot (\ln^2 x - 2 \ln x + 2) + C$$