

Určete definiční obory a spočítejte derivace:

$$(x+1)^2 > 0 \quad \checkmark$$

$$\frac{(x+1)^2}{x^2+1} \geq 0 \rightarrow x^2+1 \geq 0 \quad \checkmark$$

$$x^2 \geq -1 \quad \checkmark$$

Df: $\mathbb{R}$

$$1) \sqrt{\frac{(x+1)^2}{x^2+1}} = \left(\left(\frac{(x+1)^2}{x^2+1} \right)^{\frac{1}{2}} \right)' = \frac{1}{2} \cdot \left(\frac{(x+1)^2}{x^2+1} \right)^{-\frac{1}{2}} \cdot \frac{((x+1)^2)' \cdot (x^2+1) - (x+1)^2 \cdot (x^2+1)'}{(x^2+1)^2} =$$

$$= \frac{1}{2} \cdot |x+1|^{-1} \cdot \frac{2(x+1) \cdot (x^2+1) - (x+1)^2 \cdot 2x}{(x^2+1)^2} = \frac{|x+1|^{-1}}{(x^2+1)^{\frac{3}{2}}} \cdot \frac{(x+1) \cdot (x^2+1) - (x+1)^2 \cdot x}{(x^2+1)^2} =$$

$$\frac{x^3+x^2+x+1 - x^3-2x^2-x}{(x^2+1)^{\frac{3}{2}} \cdot |x+1|} = \frac{1-x^2}{(x^2+1)^{\frac{3}{2}} \cdot |x+1|}$$

2) $\ln(\sin(e^x))'$

$\sin(e^x) > 0$

$\sin(b) > 0 : b \in (0 + 2k\pi, \pi + 2k\pi) \Rightarrow e^x \in (0 + 2k\pi, \pi + 2k\pi)$

Df: $x \in (\ln(2k\pi), \ln(\pi + 2k\pi)) : k \in \mathbb{Z}$

$e^x > 2k\pi$

$\wedge$

$e^x < \pi + 2k\pi$

$\ln(e^x) > \ln(2k\pi)$

$\ln(e^x) < \ln(\pi + 2k\pi)$

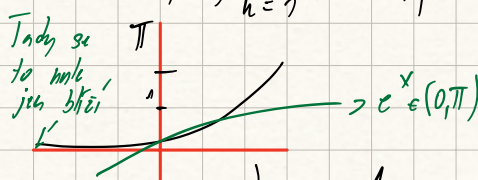
$x \cdot \ln e > \ln(2k\pi)$

$x \cdot \ln e < \ln(\pi + 2k\pi)$

$x > \ln(2k\pi)$

$x < \ln(\pi + 2k\pi)$

$D = (-\infty, \pi) \bigcup_{k=1}^{\infty} (\ln(2k\pi), \ln(\pi + 2k\pi))$



$$\ln(\sin(e^x))' = \frac{1}{\sin(e^x)} \cdot (\sin(e^x))' \cdot (e^x)' = \frac{1}{\sin(e^x)} \cdot \cos(e^x) \cdot e^x = \frac{\cos(e^x)}{\sin(e^x)} \cdot e^x = e^x \cdot \cot(e^x)$$

3) $(x \cos x + \sin(2x^2))'$

$$= (x \cdot \cos x)' + (\sin(2x^2))' = x' \cdot \cos x + x \cdot (\cos x)' + (\sin(2x^2))' \cdot (2x^2)' =$$

$$= \cos x - x \cdot \sin x + \cos(2x^2) \cdot 4x$$

Df: $\mathbb{R}$

$$4) \left(\frac{3^x + x^3}{3^x - x^3} \right)'$$

$$Df: 3^x - x^3 \neq 0$$

$$3^x \neq x^3$$

$$\log_3 3^x \neq \log_3 x^3$$

$$x \log_3 3 \neq 3 \log_3 x$$

$$x \neq 3 \cdot \log_3 x$$

$$Df: \frac{x}{\log_3 x} \neq 3$$

$$\left(\frac{3^x + x^3}{3^x - x^3} \right)' = \frac{(3^x + x^3)' \cdot (3^x - x^3) - (3^x + x^3) \cdot (3^x - x^3)'}{(3^x - x^3)^2} = \frac{(3^x \cdot \ln 3 + 3x^2) \cdot (3^x - x^3) - (3^x + x^3) \cdot (3^x \ln 3 - 3x^2)}{(3^x - x^3)^2}$$

$$= \frac{\cancel{3^x \cdot 3^x \cdot \ln 3} + \underline{3x^2 \cdot 3^x} - \underline{x \cdot 3^x \cdot \ln 3} - \cancel{x \cdot 3x^2} - \cancel{3^x \cdot 3^x \cdot \ln 3} + \underline{3^x \cdot 3x^2} - \cancel{x \cdot 3^x \cdot \ln 3} - \cancel{x \cdot 3x^2}}{(3^x - x^2)^2} =$$

$$= \frac{2 \cdot 3x^2 \cdot 3^x - 2 \cdot x^3 \cdot 3^x \cdot \ln 3}{(3^x - x^2)^2} = \frac{2 \cdot (3^{x+1} \cdot x^2) - 2 \cdot (3^x \cdot x^3 \cdot \ln 3)}{(3^x - x^2)^2}$$

$$5) x^{\sin x} e^{-x^2}$$

Df: $\mathbb{R}$

$$(e^{\sin x \ln x})' = e^{\sin x \ln x} \cdot (\sin x \cdot \ln x)' = e^{\sin x \ln x} \cdot \left(\cos x \cdot \ln x + \frac{\sin x}{x} \right) =$$

$$\left(x^{\sin x} \cdot e^{-x^2} \right)' = \left(x^{\sin x} \right)' \cdot e^{-x^2} + x^{\sin x} \cdot (e^{-x^2})' =$$

$$= x^{\sin x} \cdot \left(\cos x \cdot \ln x + \frac{\sin x}{x} \right)$$

$$= x^{\sin x} \cdot \left(\cos x \cdot \ln x + \frac{\sin x}{x} \right) \cdot e^{-x^2} + x^{\sin x} \cdot e^{-x^2} \cdot (-2x)$$

$$\begin{aligned} f: e^x \\ g: -x^2 \\ e^x \cdot -2x = e^{-x^2} \cdot -2x = -2x \cdot e^{-x^2} \end{aligned}$$

$$= x^{\sin x} \cdot e^{-x^2} \left(\cos x \cdot \ln x + \frac{\sin x}{x} - 2x \right)$$