Spočítejte primitivní funkci:

1) $\int x^{\frac{3}{2}} + x^{\frac{2}{3}}\,dx = \int x^{\frac{3}{2}}\,dx + \int x^{\frac{2}{3}}\,dx = \frac{2}{5}x^{\frac{5}{2}} + \frac{3}{5}x^{\frac{5}{3}} + c$

2) $\int \frac{(1-x)^3}{x}\,dx = \int \frac{-x^3+3x^2-3x+1}{x}\,dx = \int -x^2 + 3x - 3 + \frac{1}{x}\,dx =$

$-\int x^2 dx + \int 3x\,dx - \int 3\,dx + \int \frac{1}{x}\,dx = -\frac{x^3}{3} + \frac{3x^2}{2} - 3x + \log|x| + c$

$\int 2\cdot\frac{1}{x-1}$

3) $\int \frac{x+1}{x-1}\,dx = \int \frac{x-1}{x-1} + \frac{2}{x-1}\,dx = \int 1\,dx + \int \frac{2}{x-1}\,dx = x + 2\log|x+1| + c$

4) $\int \operatorname{cotg}(x)\,dx = \int \frac{\cos x}{\sin x}\,dx \quad \begin{matrix} t = \sin x \\ dt = \cos x\,dx \end{matrix} = \int \frac{1}{t}\,dt = \log|t| = \log|\sin x| + c$

5) $\int \underset{f}{x^2}\,\underset{g'}{e^{-x}}\,dx = \quad \begin{matrix} f = x^2 & g' = e^{-x} \\ f' = 2x & g = -e^{-x} \end{matrix} \quad = -e^{-x}x^2 + 2\cdot\int \underset{g'}{e^{-x}}\,\underset{f}{x}\,dx = \quad \begin{matrix} f = x & g' = e^{-x} \\ f' = dx & g = -e^{-x} \end{matrix}$

$-e^{-x}x^2 + 2\cdot(-e^{-x}\cdot x + e^{-x}) = -e^{-x}x^2 - 2e^{-x}x - 2e^{-x} =$

$-e^{x}\cdot(x^2 - 2x - 2) + c$

6) $\int \frac{2x}{1-x^2}\,dx = \int \frac{2x}{(1-x)\cdot(1+x)} =$

$\frac{\alpha}{(1-x)} + \frac{\beta}{(1+x)} = \frac{\alpha + \alpha x + \beta - \beta x}{(1-x)\cdot(1+x)}$

$\alpha x - \beta x = 2x$

$\alpha + \beta = 0$

$-\beta x - \beta x = 2x$

$\boxed{\begin{matrix}\beta = -1 \\ \alpha = 1\end{matrix}}$

$= \int \frac{1}{1-x} - \frac{1}{1+x}\,dx = \int \frac{1}{1-x}\,dx - \int \frac{1}{1+x}\,dx = -\int \frac{1}{x-1}\,dx - \int \frac{1}{x+1}\,dx$

$= -\log|x-1| - \log|x+1| + c$

7) $\int \frac{x+1}{x^2+5x+6}\,dx =$

$\frac{x+1}{(x+3)\cdot(x+2)} \qquad \frac{\alpha}{(x+3)} + \frac{\beta}{(x+2)}$

$\alpha x + 2\alpha + \beta x + 3\beta$

$2\alpha + 3\beta = 1 \quad 2 - 2\beta + 3\beta = 1 \rightarrow \beta = -1$

$\alpha + \beta = 1 \quad \alpha = 1 - \beta$

$\alpha - 1 = 1$

$\alpha = 2$

$\beta = -1$

$\rightarrow \int \frac{2}{x+3} - \frac{1}{x+2}\,dx = 2\cdot\int \frac{1}{x+3}\,dx - \int \frac{1}{x+2}\,dx$

$= 2\cdot\log|x+3| - \log|x+2| + c$

$\frac{3x}{(x+2)\cdot(x-1)} = \frac{\alpha}{(x+2)} + \frac{\beta}{(x-1)}$

$\alpha x - \alpha + \beta x + 2\beta = 3x$

8) $\displaystyle\int \frac{x^2-2x-2}{x^2+x-2}\,dx = \int \frac{x^2+x-2}{x^2+x-2} - \frac{3x}{x^2+x-2}\,dx = \int 1\,dx - \int \frac{3x}{x^2+x-2}\,dx =$

$\alpha+\beta=3$

$-\alpha+2\beta=0$

$2\beta=\alpha$

$2\beta+\beta=3$

$\beta=1$

$\alpha=2$

$\displaystyle = x - \int \frac{2}{(x+2)} + \frac{1}{(x-1)}\,dx = x - 2\cdot\int\frac{1}{x+2}\,dx + \int\frac{1}{x-1}\,dx =$

$= x - 2\cdot \log|x+2| + \log|x-1| + c$

9) $f \quad g'$

$f = x^2 \qquad g' = \cos x$

$df = 2x\,dx \qquad g = \sin x$

$f \quad g' \qquad df = 2dx \quad g = -\cos x$

$\displaystyle\int x^2\cos x\,dx = x^2\cdot\sin x - \int 2x\cdot\sin x\,dx = x^2\cdot\sin x - \left(2x\cdot(-\cos x) - \int 2\cdot(-\cos x)\right)$

$= x^2\cdot\sin x + 2x\cdot\cos x - 2\sin x + c \quad = (x^2-2)\sin x + 2x\cos x + c.$

10) $t = x^{\frac{1}{2}} \qquad dt = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

$\displaystyle\int \frac{1}{(x+1)\sqrt{x}}\,dx = 2\cdot\int\frac{1}{t^2+1}\,dt = 2\cdot\arctg(t) = 2\cdot\arctg(\sqrt{x}) + c$

$x^{\frac{3}{2}} + x^{\frac{1}{2}}$