NPGR036 Homework 1

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March 6, 2023

General information

There are three tasks totalling 100 points concerning coloured photo creation and colour quantisation. The third task is theoretical and should be completed on a piece of paper or in a formula-friendly writing software. On the other hand, the first two tasks are programming exercises and they will be evaluated using evaluation script `evaluator.py`. The evaluation script is supplied together with the assignment so that you can try that your solution works as expected.

1 Bayer demosaicing

The first task is to reconstruct a coloured image from a Bayer encoding of an image produced by cameras. There are four types of encoding masks titled `BGGR`, `RGGB`, `GRBG` and `GBRG` according to the 2x2 region of the Bayer pattern. Your task is to compute demosaicing of the image using all four masks and return these four images. You should demosaic individual pixels from their 3x3 neighbourhood by simple averaging if the value is not present.

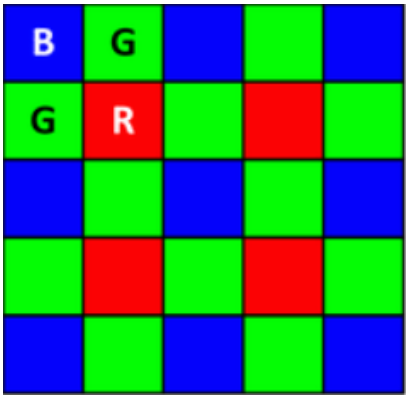

Figure 1: Example of the Bayer BGGR mask.

Your results will be compared against the implementation in OpenCV, which computes almost the same thing as we request. The results from OpenCV are not entirely correct due to some performance optimisations. Therefore, there is a tolerance (2.5) of the accepted difference of your solution. The following lines show examples of running the evaluator for the Bayer task.

```
python .\evaluator.py --test=bayer --image=data/Lighthouse_bggr.png --tol=2.5
python .\evaluator.py --test=bayer --image=data/04.jpg_rggb.pgm --tol=2.5
```

Verify (visually) that the Bayer pattern written in the name of the image file is truly the correct one.

2 Median cut

The second task is to implement the median cut algorithm for colour quantisation of a given image. The exact implementation details are not as important, and therefore, there is no error value. Instead, the evaluation script will show you the original image, the groups of pixels which were reduced to a single colour and the reconstruction of the image using your palette 2.

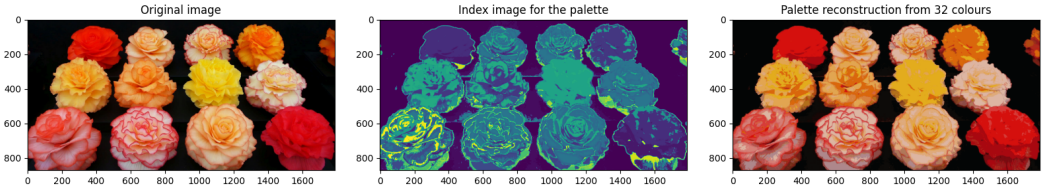

Figure 2: Example of median cut results.

Your solution should return two numpy arrays, one is a Kx3 palette of colours computed by your algorithm and the other is an array with the same resolution as the original image filled with indices into your palette. You have to keep track which pixels are grouped into which groups throughout the computation. If the image contains less colours than the specified amount then return the colours present in the image (and compute the index image).

The following line shows an example of running the evaluator for the median cut task.

```
python .\evaluator.py --test=mediancut --image=data/im36.jpg --num_colors=32
```

3 Quantisation MSE

Based on the formula for minimisation of the mean squared error of quantisation 1, derive that the conditions of the closest neighbour and centroid hold. We denote the number of pixels as n.

$$E = \frac{1}{n} \sum_i d^2(x_i, Q(x_i)) \tag{1}$$

Let us assume the following notation: quantised regions are $\{R_i | i = 1, ..., N\}$, the elements of the palette are $\{c_i | i = 1, ..., N\}$, image colours are x and the quantisation function is $Q(x) = c_i$

The condition of the closest neighbour is that for each colour, its assigned quantised colour is not further than any other quantised colour. Formally, for a given palette P, the optimal regions $\{R_i | i = 1, ..., N\}$ satisfy the constraint 2:

$$R_i \subset \{x | d(x, c_i) \leq d(x, c_j) \forall j\} \tag{2}$$

and therefore 3:

$$Q(x) = c_i \leftrightarrow d(x, c_i) \leq d(x, c_j) \forall j \tag{3}$$

The centroid condition states that the arithmetic mean of a colour region (Equation 5 where x_k belongs to R) is the optimal quantisation colour for MSE minimisation. Formally, for the given regions $\{R_i | i = 1, ..., N\}$, the elements of the palette satisfy the condition 4:

$$c_i = cent(R_i) \tag{4}$$

$$cent(R) = \frac{1}{|R|} \sum_k^{|R|} x_k \tag{5}$$

You should prove equations 3 and 4. It can be a direct proof or a proof by contradiction or any other method you choose. Write your proofs on a piece of paper or in a formula-friendly writing software and submit their photo/PDF.