

Abstract approach to templates

► A template is a function evaluated by the compiler

► Its arguments may be:

- integral constants

```
template< std::size_t N> /*...*/;
```

- types

```
template< typename T> /*...*/;
```

→ tohle je nejčastější

- other templates (voilà: functional programming)

```
template< template< /*...*/> class T> /*...*/;
```

Abstract description of templates in C++

► A template is a function evaluated by the compiler

► Its return value may be one of:

- an integral constant

- indirectly (convention: static member constant named "value")

```
template< /*...*/> class F { static constexpr int value = /*...*/; };
```

- directly [C++14] (convention: named "F_v" if defined as F::value)

```
template< /*...*/> constexpr int F_v = /*...*/;
```

- a type

- a class - directly

```
template< /*...*/> class F { /*...*/};
```

- any type indirectly (convention: member named "type")

```
template< /*...*/> class F { typedef /*...*/ type; };
```

- any type directly [C++11] (convention: named "F_t" if defined as F::type)

```
template< /*...*/> using F_t = /*...*/;
```

- a template

- indirectly

```
template< /*...*/> class X { template< /*...*/> using type = /*...*/; };
```

- a run-time function (a function template or a static member function of a class template)

Abstract description of templates in C++

- ▶ A template is a function evaluated by the compiler
 - ▶ Its arguments may be:
 - integral constants
 - types
 - templates, i.e. compile-time functions
 - ▶ Its return value may (directly or indirectly) be one of:
 - an integral constant
 - a type
 - a template, i.e. a compile-time function
 - a run-time function
- ▶ A template may also "return" a "compile-time structure":

```
template< /*...*/> class F {
```

```
    using A = /*...*/;
```

```
    static const int B = /*...*/;
```

```
    static int C(int x, int y) { /*...*/ }
```

```
};
```

takhle můžeme chýpnout funkci, co má nějakou hodnotu

- ▶ There is a *compile-time programming language* inside C++
 - ▶ The language can operate on the following atomic "values":
 - integral constants
 - types
 - run-time functions
 - ▶ The operations available are
 - integral operators of C++ (plus part of standard library marked *constexpr*)
 - type constructions (creating pointers, references, arrays, classes, ...)
 - defining a new run-time function (which may call others, including compile-time "values")
 - ▶ The "values" may be combined into "structures"
- ▶ The compile-time language is *functional*
 - ▶ no variables which could change their value
 - ▶ "compile-time functions" (i.e. class templates) are first-class "values"

- ▶ **constexpr functions** — *> a proměnné jde o konstantu vyhodnocenou překladačem*
 - ▶ The compiler will *interpret* the code of the function during compilation
 - ▶ Limitations are gradually being lifted
 - C++11: A constexpr function must consist of single return statement
 - C++14: No virtual functions, no dynamic allocation, no goto/asm/throw/try-catch
 - C++17: No virtual functions, no goto/asm/throw/try-catch
 - C++20: No goto/asm, no static variables, few other limitations
 - ▶ No access to run-time objects allowed *↳ můžeme dynamicky alokovat, ale musíme mít možnost po sobě uklídit*
Nelze si ale připravit při překládání data pro runtime...
 - proto má std::string i constexpr destructor
- ▶ **constexpr functions, variables:**
 - ▶ Must be evaluated by compiler when used in template arguments etc.
 - In this case, any objects must have their lifetime confined within the evaluation
 - ▶ May be evaluated by compiler as an optimization elsewhere
 - ▶ [C++20] **constexpr**, **constexpr**: every call/init must return a constant
- ▶ Cf. **const** variables, references, pointers, functions:
 - ▶ Compiler prohibits run-time modification (via this access path only)

► constexpr functions vs. template metaprogramming

- constexpr function always obey the rules for run-time functions
 - Types can not be manipulated by constexpr functions
 - Expressions must be statically typed
 - A loop can not work with elements of different types

```
using T = std::tuple<int,double,string>;
```

```
/*constexpr*/ T for_each(T p)
```

```
{
```

```
    for (auto && x : p)          // ILLEGAL - std::tuple does not have begin()/end()
```

```
        x += 'A';              // ILLEGAL - x is not statically typed here
```

```
    return p;
```

→ v každém iteraci je to jiný typ, to nikdy překladač nedá

```
}
```

- Using templates, implementation of such loops is possible, although tricky

- ▶ There is a *compile-time programming language* inside C++
- ▶ The compile-time language is *functional*
- ▶ There are *Prolog*-like features
 - ▶ A "compile-time function" (i.e. a template) may be defined by several independent rules (using partial specialization)

```
template< typename T, int N> class F;
```

```
template< typename T> class F<T, 0> { /*...*/ };
```

```
template< typename U, int N> class F<U*,N> { /*...*/ };
```

```
template< typename P> class F<X<P>, 1> { /*...*/ };
```

*Jistí forma prologové unifikace
– podle specializace zvolí impl.*

- There is unification of terms representing type arguments
- But unlike Prolog, there is no priority or try-next-if-failed mechanism
- ▶ With [C++20] concepts we can even emulate Prolog '!'
 - Before [C++20], it required decltype(), function templates, and SFINAE

- ▶ There is a *compile-time programming language* inside C++
- ▶ C++ types form a complex universe of "compile-time values"
 - ▶ Classes (class/struct/union) are compared **by name** — *→ dvě stejné struktury se stejným obsahem nejsou „stejné“*
 - New unique "values" are generated by each occurrence of structure in source code
 - Empty structures may be employed as unique opaque values (usually called *tags*)
 - ▶ Other type constructs (*,&,&&,[],()) are compared **by contents**
 - This allows unification in partial specializations — *→ pointer bude i po přímazení furt stejný...*
 - ▶ Instances of the same class template are considered equal if their arguments are equal
 - ▶ Aliases created by typedef/using are considered equal to their defining types

- ▶ There is a *compile-time programming language* inside C++
- ▶ The output of the compile-time program is a run-time program
 - ▶ *Compile-time functions* may return *run-time types* and *run-time functions*
 - They may also generate static_assert and other error messages
 - ↳ i když tam error bude, tak nemůže runtime, do kterého bych to poslal!
- ▶ Elements used only during compile-time
 - ▶ integral constants
 - ▶ structures containing types, constants, static functions
 - often called *policy class*
 - even empty structures are important as *tags*
 - ▶ templates perceived as compile-time functions
 - those returning policy classes are often called *traits*
- ▶ Compile-time elements converted to run-time representations
 - ▶ C++ types
 - ▶ run-time functions

► Compile-time arithmetics

C++11: <ratio>

- Rational numbers

```
template< intmax_t n, intmax_t d = 1> struct ratio;
```

- Example

```
using platform = ratio_add<ratio<9>, ratio<3,4>>;
```

// 9 // 3/4

→ jde pouze o název, nemá žádnou hodnotu

```
static_assert( ratio_equal< platform, ratio< 975, 100>>::value, "?");
```

- Evaluated by the compiler
 - Including computing the greatest common divisor!
- No longer needed because of stronger constexpr functions



Metaprogramming

"Nõõim si neekim generant mij runtimonj program"

Example

- Type transformation

- Goal: Transform, e.g., this triplet ...

`using my_triplet = std::tuple< int, my_class, double>;` *→ označuje něco jako funkci, as dostane typ a vrátí jiný typ*

- ... into the triplet `std::tuple< F(int), F(my_class), F(double)>`

- What it is good for?

- column-stores:

↳ například na vstupu je typ seznamu, na výstupu je typ vektoru

`std::tuple< std::vector<int>, std::vector<my_class>, std::vector<double>>`

- passing of references:

`std::tuple< int &, my_class &, double &>`

Example

- Type transformation

- Goal: Transform, e.g., this triplet ...

```
using my_triplet = std::tuple< int, my_class, double>;
```

- ... into the triplet `std::tuple< F(int), F(my_class), F(double)>`

- ▶ How to invoke the transformation?

- Standard C++ library uses the “`::type`” convention:

```
using my_transformed_triplet = type_transform< my_triplet, F>::type;
```

- In case of dependent types, „`typename`“ is required

```
template< typename some_triplet> class X {
```

```
    using transformed_triplet = typename type_transform< some_triplet, F>::type;
```

```
};
```

- C++14 adds the “`_t`” convention:

```
using my_transformed_triplet = type_transform_t< my_triplet, F>;
```

- No `typename` required

Example

- Type transformation

- Goal: Transform, e.g., this triplet ...

```
using my_triplet = std::tuple< int, my_class, double>;
```

- ... into the triplet `std::tuple< F(int), F(my_class), F(double)>`

```
using my_transformed_triplet = type_transform_t< my_triplet, F>;
```

- ▶ How can we specify the F "function" for `type_transform_t`?

- A. Directly:

ai nůle hlouběji bude použít' takové argumenty s <...>

```
using my_reference_triplet = type_transform_t< my_triplet, std::add_lvalue_reference_t>;
```

- Failed before C++17 because `std::add_lvalue_reference_t` is not a class template

```
using my_vector_triplet = type_transform_t< my_triplet, std::vector>;
```

- Failed before C++17 because `std::vector` has 3 arguments but `type_transform_t` expects 1

Template template arguments

► Template template arguments before C++17

```
template< template< typename> class P>
```

```
class example { using a_class = P<int>; }
```

- The actual argument was required to be a class/struct template

```
template< typename T> using A1 = std::vector< T>;
```

```
using my_type1 = example< A1>;
```

- Failed because A1 is an alias template
- The arguments of the actual argument were required to exactly match the arguments of the formal argument

```
using my_type2 = example< std::vector>;
```

- Failed because std::vector has three arguments (although two of them have default values)

► C++17 changed the behavior

- Alias templates are allowed as actual arguments, default arguments considered
- The compatibility of the actual argument has a new, hardly readable definition:
 - Formally, a template template-parameter P is at least as specialized as a template template argument A if ...
- "typename" may be used instead of "class" in the formal argument declaration

► Both the examples work in C++17

Example

► How can we specify the F function for type_transform_t?

A. Directly (requires C++17):

- The argument is a template class/struct or template alias

```
using my_reference_triplet = type_transform_t< my_triplet, std::add_lvalue_reference_t>;
```

```
using my_vector_triplet = type_transform_t< my_triplet, std::vector>;
```

B. Indirectly, using the well-known "::type" convention:

- The argument is a template class/struct containing an alias member named "type"

```
using my_reference_triplet = type_transform_t< my_triplet, std::add_lvalue_reference>;
```

```
template< typename T> struct add_vector { using type = std::vector< T>; };
```

```
using my_vector_triplet = type_transform_t< my_triplet, add_vector>;
```

C. Indirectly, using a nested template

- The argument is a class/struct containing a template alias member named "type"
- This is an equivalent of a functor, in the realm of meta-functions returning types

```
struct add_vector { template< typename T> using type = std::vector< T>; };
```

```
using my_vector_triplet = type_transform_t< my_triplet, add_vector>;
```

položka smlouvením jménem, šablónování

Example

- ▶ The implementation of `type_transform` must match the selected convention

- A. Directly (requires C++17):

- The argument is a template class/struct or template alias

```
template< typename T, template< typename> typename F>
struct type_transform {
    using example_usage = F< int>;
};
```

- B. Indirectly, using the well-known “`::type`” convention:

- The argument is a template class/struct containing an alias member named “`type`”

```
template< typename T, template< typename> class F>
struct type_transform {
    using example_usage = typename F< int>::type;
};
```

- c. Indirectly, using a nested template

- The argument is a class/struct containing a template alias member named “`type`”

```
template< typename T, typename F>
struct type_transform {
    using example_usage = typename F::template type<int>;
};
```

- ▶ `type_transform_t` is a wrapper with the same arguments as `type_transform`

Example

► The full implementation (C++17)

- General template struct declaration

```
template< typename T, template< typename> typename F>
```

```
struct type_transform;
```

- Partial specialization for std::tuple

- The partial specialization allows to extract the type list L from std::tuple< L...>
- A type list can exist only as a template argument – accessible only inside such a template

```
template< typename ... L, template< typename P> class F>
```

```
struct type_transform< std::tuple< L ...>, F>
```

```
{
```

```
    using type = std::tuple< F< L> ...>;
```

```
};
```

- We may also implement specializations for std::pair or std::array

- Wrapper alias template

```
template< typename T, template< typename> typename F>
```

```
using type_transform_t = typename type_transform< T, F>::type;
```

- In the other two conventions, more complex type-ids are used instead of F<L>

*takže třetí vector jde, protože lze volat s
jedním argumentem*

*→ takže říká, že to musí být šablona
vyvolatelů s jedním parametrem*

Iterating through n-tuple elements

Example

► Working with tuple values

```
using my_triple = std::tuple< int, my_class, double>;
```

```
using my_transformed_triple = type_transform_t< my_triple, std::vector>;
```

```
my_triple a;
```

```
my_transformed_triple c;
```

```
auto my_value_function = /*...*/;
```

```
static_transform(a, c, my_value_function);
```

*chci, aby mi to transformovalo int -> std::vector<int>,
my_class -> std::vector<my_class>...*

- `static_transform` shall be an equivalent of `std::transform` for tuples

- The intent is to apply `f` to every element of `x` and store the results in `r`
- Implementation like this will **not** compile:

```
template< typename T1, typename T2, typename F>
```

```
void static_transform( T1 && x, T2 && r, F f)
```

```
{ for ( std::size_t i = 0; i < std::tuple_size_v< T1>; ++i)
```

```
    std::get<i>(r) = f( std::get<i>(x));
```

```
}
```

takle nepujde překladačem

↳ tedy ale potřebuji konstantu, což vícená proměnná for-cyklu neumí.

- `my_value_function` is a polymorphic functor

- applied to arguments of different types, may return different types
- in this case, it transforms single values into vectors

*Napukl chci kód pro každý typ
speciálně.*

Nikdy to nebude jedna vnitřní instance, která by se volala na všechny typy.

► Polymorphic functors

- applied to arguments of different types, may return different types
- in this case, it transforms single values into vectors

► The simplest case is a polymorphic lambda (C++14)

```
auto my_value_function = [](auto && x){  
    return std::vector</*???*/>( 1, x);  
};
```

- Problem: How to derive the type for the vector?

tohle nepostichujin

```
std::vector<std::remove_cv_t<std::remove_reference_t<decltype(x)>>> X
```

- Problem: The argument x shall be passed using `std::forward</*???*/>( x)`

► Since C++20, lambda may contain explicit template arguments

```
auto my_value_function = []<typename T>(T && x){  
    return std::vector<std::remove_cvref_t<T>>( 1, std::forward<T>(x));  
};
```

↳ tohle mi vyhoví std::vector<T> z nějakého typu T, který jí předám referencí

místo toho

► Polymorphic functors

- applied to arguments of different types, may return different types
- in this case, it transforms single values into vectors

► Explicitly defined functor

```
struct my_value_functor {  
    template< typename T1>  
    std::vector<T1> operator()( T1 && x) const  
    { return std::vector<T1>( 1, std::forward<T1>(x));  
    }  
};  
  
auto my_value_function = my_value_functor{};
```

- BEWARE: It does NOT work correctly!
- If the actual argument is an lvalue of type T, T1 would be T &
 - Reference collapsing rules apply
- The result type would be a vector of references!

► The problem encountered here is quite frequent and often forgotten!

► Polymorphic functors

- applied to arguments of different types, may return different types
- in this case, it transforms single values into vectors

► Explicitly defined functor – corrected implementation

```
struct my_value_functor {  
    template< typename T1>  
    auto operator()( T1 && x) const  
    { return std::vector<std::remove_cv_t<std::remove_reference_t<T1>>>  
        ( 1, std::forward<T1>(x));  
    }  
};
```

- `std::remove_reference_t` removes `&` or `&&` if present
- `std::remove_cv_t` removes `const` or `volatile` if present
- C++20: `std::remove_cvref_t` combines the two
- Using `auto` in the operator declaration avoids repeating the ugly type-id

► Polymorphic functors

- applied to arguments of different types, may return different types
- in this case, it transforms single values into vectors

► Polymorphic lambda – corrected implementation

```
auto my_value_function = [](auto && x){
```

```
    using x_t = decltype(x);
```

```
    using e_t = std::remove_cv_t< std::remove_reference_t< x_t>>;
```

```
    return std::vector<e_t>( 1, std::forward<x_t>(x));
```

```
};
```

↳ takže je trochu podvod, ale výsledek je stejný...

- `decltype(x)` is the type of `x`, always a (lvalue or rvalue) reference
- `std::forward` is used differently from the usual practice, but it works:
 - Our code is equivalent to

```
template< typename U> auto f(U && x) { /*...*/ std::forward<U &&>(x) /*...*/ }
```

- `std::forward<T>(x)` is by definition equivalent to `static_cast<T&&>(x)`
- Therefore, `std::forward<U&&>` is equivalent to `std::forward<U>` due to reference collapsing

Example

► Iterating through n-tuple elements

- A non-working implementation

```
template< typename A, typename R, typename F>
```

```
void static_transform(A && a, R && r, F f)
```

```
{
```

```
    using atuple = std::remove_cv_t< std::remove_reference_t< A>>;
```

```
    using rtuple = std::remove_cv_t< std::remove_reference_t< R>>;
```

```
    static constexpr auto asize = std::tuple_size_v< atuple>;
```

```
    static constexpr auto rsize = std::tuple_size_v< rtuple>;
```

```
    static_assert(asize == rsize, "tuples must be of equal length");
```

```
    for ( std::size_t i = 0; i < std::tuple_size_v< T1>; ++i)
```

```
        std::get<i>(r) = f( std::get<i>(x));
```

```
}
```

dělostřelecký přípust

— > takhle funkt. nelze přeložit

- `std::get<i>` requires constant `i`
- The compiler must be able to instantiate `f()` for the correct types

Example

- ▶ Iterating through n-tuple elements
 - Reformulated using a compile-time version of `std::for_each`
 - `static_for_each_index` calls `transform_ftor` for each index in the given range

```
template< typename A, typename R, typename F>
void static_transform(A && a, R && r, F f)
{
    using atuple = std::remove_cv_t< std::remove_reference_t< A>>;
    using rtuple = std::remove_cv_t< std::remove_reference_t< R>>;
    static constexpr auto asize = std::tuple_size_v< atuple>;
    static constexpr auto rsize = std::tuple_size_v< rtuple>;
    static_assert(asize == rsize, "tuples must be of equal length");

    static_for_each_index< 0, rsize>(
        transform_ftor< A, R, F>( also arguments from transform
            std::forward< A>(a), std::forward< R>(r), std::move(f)));
}
```

Example

▸ static_for_each_index - interface

```
template< std::size_t b, std::size_t e, typename F>  
void static_for_each_index(F f);
```

konstanty oddělené

- Logically, it shall call $f(i)$ for every i in $[b, e)$
 - Problem: i must be a constant inside f
 - The compiler must create a separate instantiation for every $f(i)$
 - Problem: $f<i>()$ does not work because f is a variable, not a function
 - $f.operator()<i>()$ does not work either
 - Possible solution: call a normal member function instead of `operator()`
- ulátky závorek jsou runtimeová věc*
tohle normálně nejde...
tohle takto funguje

```
f.call<i>();
```

- Requires a functor like

```
struct ftor {  
    template< std::size_t i>  
    void call() const  
    { /*...*/ }  
};
```

Example

▸ static_for_each_index - interface

```
template< std::size_t b, std::size_t e, typename F>
```

```
void static_for_each_index(F f);
```

- Logically, it shall call $f(i)$ for every i in $[b,e)$
- Problem: i must be a constant inside f
- A better solution: pass the constant through the type of an argument

```
f(static_index<i>{});
```

_____ = multimodový prázdný tag, kterým dáváme typ

- The argument type is an empty tag class

```
template< std::size_t i> struct static_index {};
```

_____ > jednoduchý pomocný struktura

- Requires a functor like

```
struct ftor {
```

```
    template< std::size_t i>
```

```
    void operator()(static_index<i>) const
```

```
    { /*...*/ }
```

```
};
```

Example

- A better solution: pass the constant through the type of an argument
 - We can also use `std::integral_constant` as the tag class

```
template< std::size_t i>
```

```
using static_index = std::integral_constant<std::size_t, i>;
```

—> už vytvoření ve std::, nemusím dělat sám!

- With integral constant, we may even use a (C++14) lambda:

```
auto ftor = [](auto tag_value){  
    constexpr std::size_t i = tag_value;  
};
```

→ tohle je objekt typu s konverzním operátorem na `size_t`,
s přímým `constexpr`

to „i“ je schováno v typu toho „tag_value“

- `std::integral_constant` defines a conversion operator to `std::size_t` which, although not declared static, does not access the object and returns the template argument `i`
- therefore, the member function may be evaluated in constant context although `tag_value` is a run-time object
- at run-time, the `tag_value` is an empty object and most compilers will not reserve any space for it, consequently, there will be no run-time cost for dealing with this object
- tag objects shall always be passed by value – passing by reference may have a run-time cost

Example

► A functor for static_transform

```
template< typename A, typename R, typename F> struct transform_ftor
```

```
{
```

```
    transform_ftor(A && a, R && r, F f)
```

```
    : a_(std::forward<A>(a)), r_(std::forward<R>(r)), f_(std::move(f))
```

```
    {}
```

```
    template< std::size_t i> void operator()(static_index<i>) const
```

```
    {    std::get< i>(std::forward<R>(r_)) = f_( std::get< i>(std::forward<A>(a_)));
```

```
    }
```

_____ kopie malého funktoru, co dělá to transformaci typů

```
private: A && a_; R && r_; F f_;
```

```
};
```

_____ → když bych dostal r-value, tak i tedy ponechám r-value.

tohle může vypadat, že to bude
volat n-krát, tak to ve skutečnosti
je jen jednou.

► Formally, A && and R && in the constructor are NOT universal references

- However, we will supply the type arguments A, R from a context where A && and R && are universal references (and consistent with the actual arguments to the constructor)
 - Therefore, the use of forward in the constructor is correct
 - Since the member variables a_ and r_ have the same type as the arguments, they also act as if they were universal references
 - The use of forward in operator() is correct because it is invoked only once for each i
- if A && is rvalue reference, f_ may steal from it
- std::get propagates the rvalue/lvalue distinction from its argument

Example

- ▶ Static for-loop – an implementation by recursion
 - A class/struct template is required because we need partial specialization

```
template< std::size_t b, std::size_t n> struct for_each_index_helper {  
    template< typename F> static void call(F && f)  
    {  
        f(static_index< b>());  
        for_each_index_helper< b + 1, n - 1>::call( std::forward<F>(f));  
    }  
};
```

- Partial specialization stops the recursion

```
template< std::size_t b> struct for_each_index_helper< b, 0> {  
    template< typename F> static void call(F &&) {} —> konec rekurze, kde už nedělám nic  
};
```

začátek *konec* *<b; e>*

- Public wrapper

```
template< std::size_t b, std::size_t e, typename F>  
void static_for_each_index(F f)  
{  
    for_each_index_helper< b, e - b>::call(std::move(f));  
}
```

→ tedy to změním na počet, abych mohl dělat i funkci <b, 0>

- The public function receives the functor f by value, similarly to all std algorithms
- Internally, we pass it by universal reference to avoid excess copying

Example

► Static for-loop – implementation without recursion

- we need to generate the indices $\{0, \dots, n-1\}$

- C++14 library contains this:

```
template< std::size_t ... IL>
```

```
using index_sequence = std::integer_sequence<std::size_t, IL...>;
```

- `index_sequence` is an alias to a more general tag class

```
template< std::size_t N>
```

```
using make_index_sequence = /* black magic */;
```

- The black magic ensures that `make_index_sequence<N> == index_sequence< 0, 1, ..., N-1>`

► Usage:

- Use `make_index_sequence<N>` to generate a list of indices
 - The list is not accessible directly
- Use partial specialization (or a function template) as a context in which the list is visible as a template parameter pack
 - Similar to the implementation of `type_transform`; instead of a type list argument of a tuple, here we have a constant list argument of an `index_sequence`

Example

► Static for-loop – implementation without recursion

- A helper class

```
template< std::size_t b, typename S>
```

```
struct for_each_index_helper2;
```

- A partial specialization is used to access the template parameter pack L

```
template< std::size_t b, std::size_t ... L>
```

```
struct for_each_index_helper2< b, std::index_sequence< L ...> >
```

```
{   template< typename F> static void call(F & f)
```

```
    {   (f( static_index< b + L>()), ...); —> takhle mi to přehledně vytvořil n-luť.
```

```
    }
```

```
};
```

- A public wrapper

```
template< std::size_t b, std::size_t e, typename F>
```

```
void static_for_each_index(F && f)
```

```
{
```

```
    for_each_index_helper2< b, std::make_index_sequence< e - b> >::call(f);
```

```
}
```

Example

► Implementation of make_index_sequence

- A tag struct template

```
template< typename T, T ... IL> struct integer_sequence {};
```

- A helper struct to recursively generate a list of integers
 - Defines member type = integer_sequence< T, 0, 1, /*...*/, N-1, (IL+N) ...>

```
template< typename T, T N, T ... IL> struct integer_sequence_generator  
: integer_sequence_generator< T, N-1, 0, (IL+1)...> {};
```

- A partial specialization stops the recursion

```
template< typename T, T ... IL>  
struct integer_sequence_generator< T, 0, IL...> {  
    using type = integer_sequence< T, IL...>;  
};
```

- The public wrapper

```
template< std::size_t N>  
using make_index_sequence = typename integer_sequence_generator< std::size_t, N>::type;
```

- This is recursive at compile time, allowing to avoid recursion at run time
 - The compile-time complexity (N^2) may be improved to ($N \cdot \log(N)$)
 - Hint: `<IL..., (IL+sizeof...(IL))...>` doubles the length of the sequence