Convolutional Neural Networks

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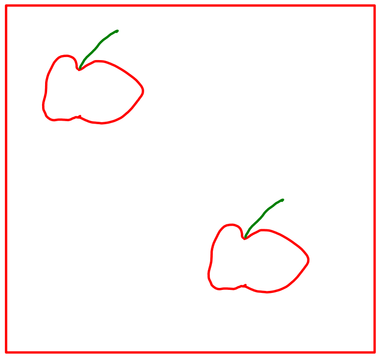
Going Deeper

Consider data with some structure (temporal data, speech, images, …).

Unlike densely connected layers, we might want:

- local interactions only;
- shift invariance (equal response everywhere).

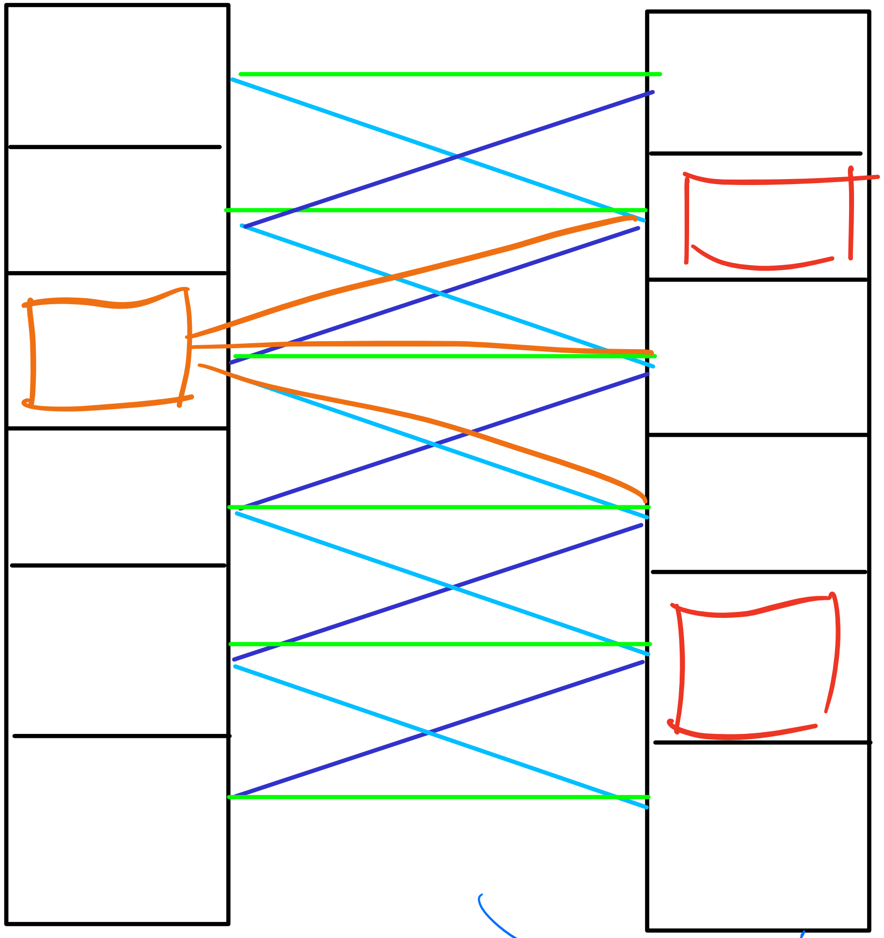
na fully connected bych se musel specificky učit, jak vypadají 2 jablka vedle sebe..



U MLP každá výstupní feature záleží na všech pixelech...

1D Convolution

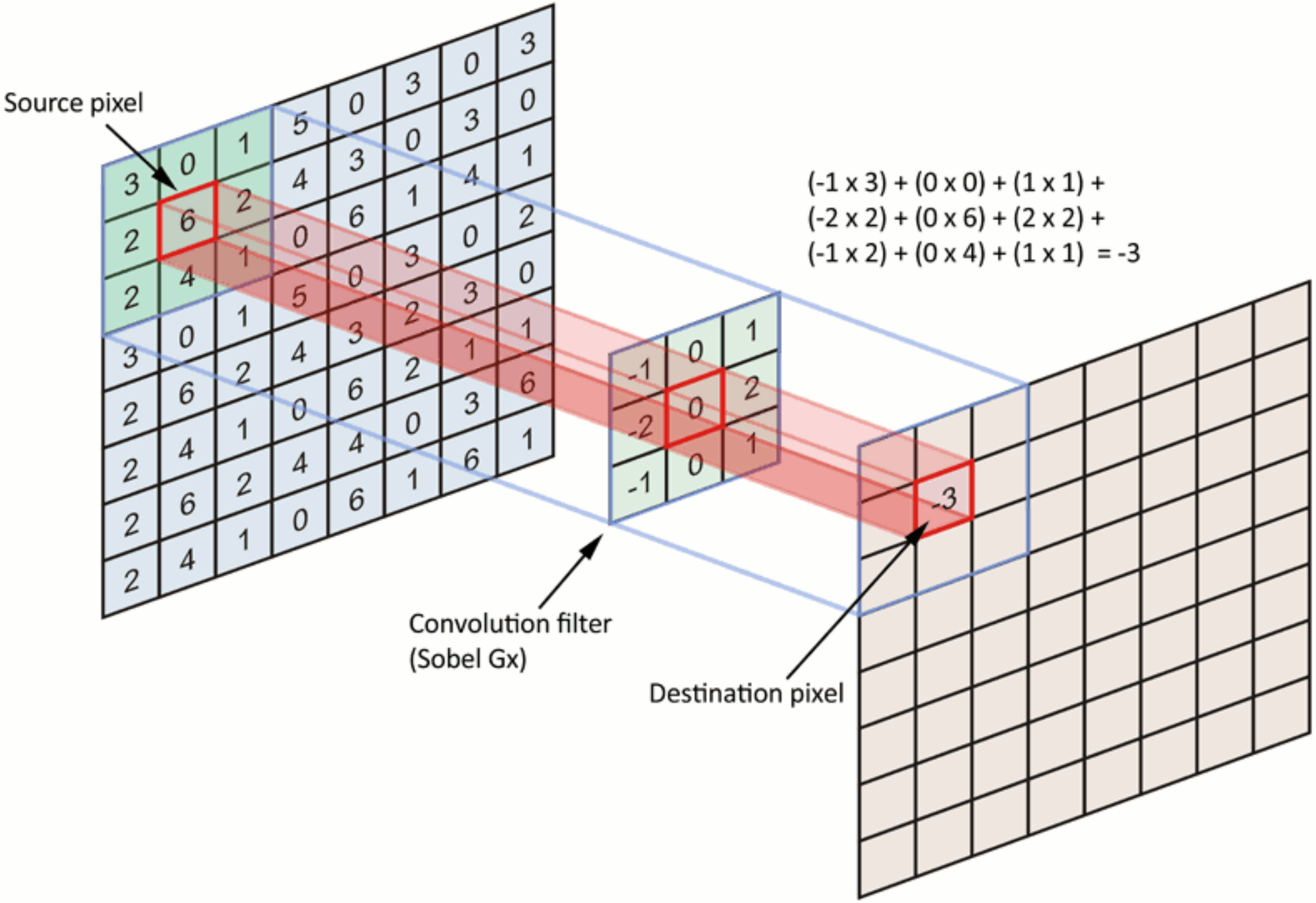
convoluční způsob:



používají stejné váhy!

> sousedi jsou definováni instancí problému

2D Convolution



https://i.stack.imgur.com/YDusp.png

2D Convolution

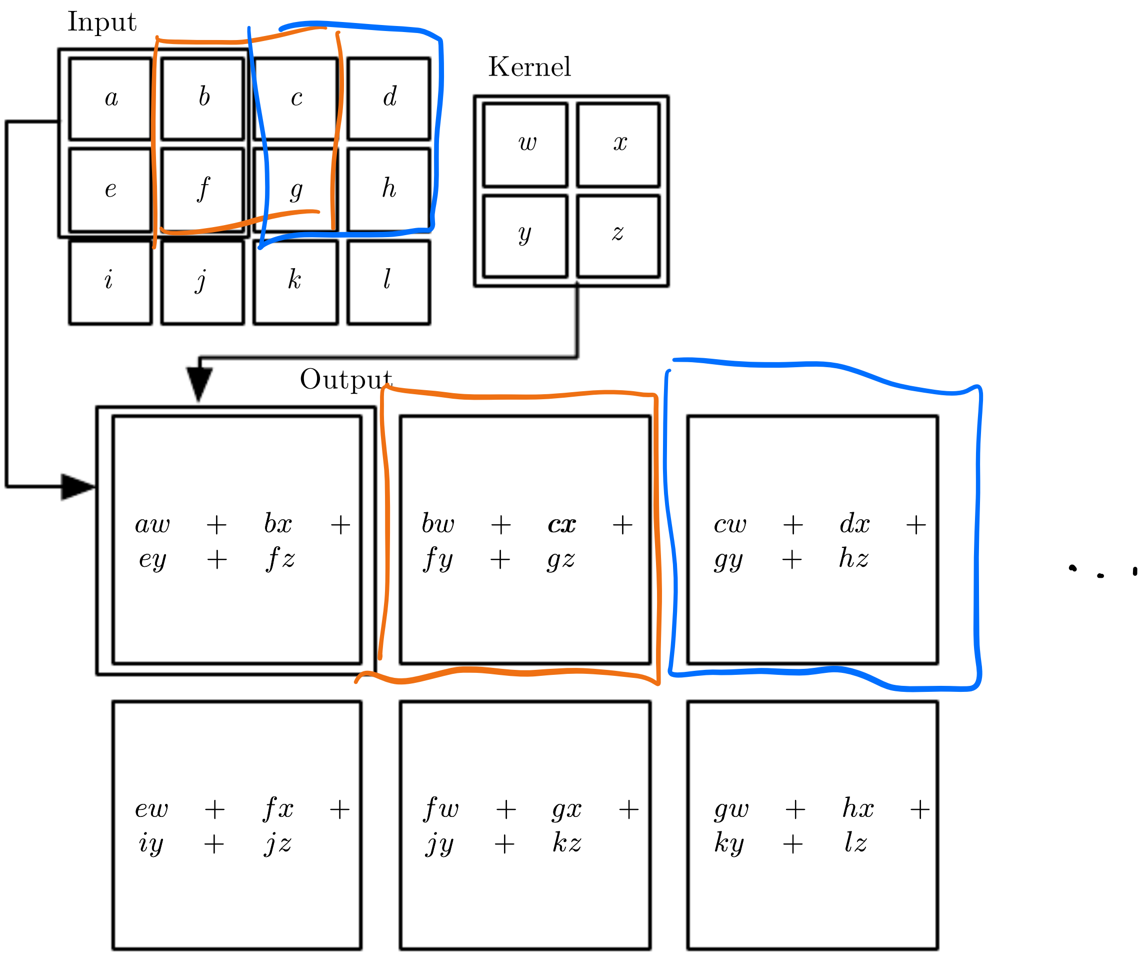
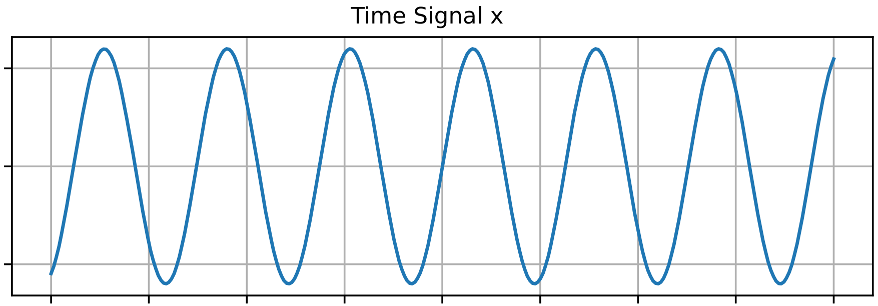


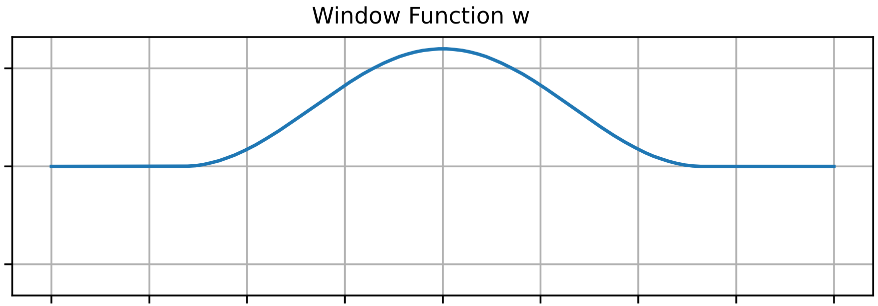
Figure 9.1 of "Deep Learning" book, https://www.deeplearningbook.org

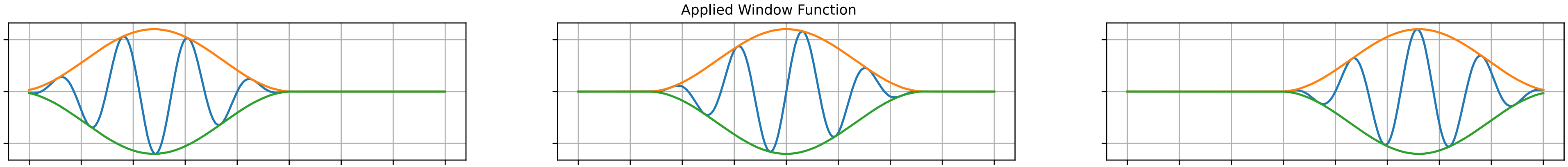
Convolution Operation

For a functions x and w, *convolution* $w * x$ is defined as

$$(w * x)(t) = \int x(t - a)w(a)\,\mathrm{d}a.$$





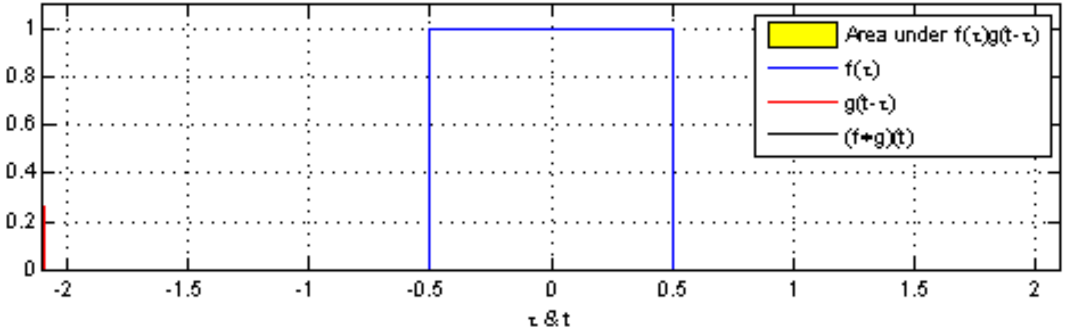


← = jak vypadá funkce na nějakém okolí

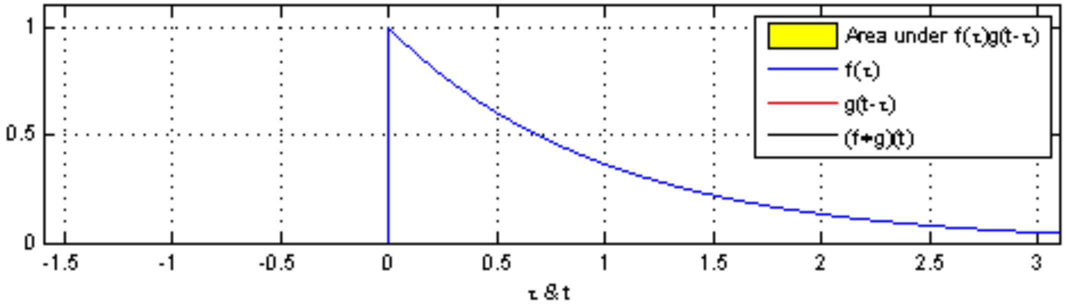
Convolution Operation

For a functions x and w, *convolution* $w * x$ is defined as

$$(w * x)(t) = \int x(t-a)w(a)\,\mathrm{d}a.$$



https://commons.wikimedia.org/wiki/File:Convolution_of_box_signal_with_itself2.gif



https://commons.wikimedia.org/wiki/File:Convolution_of_spiky_function_with_box2.gif

Convolution Operation

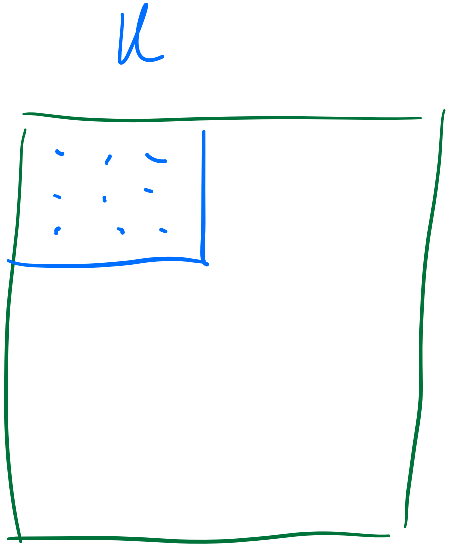
For a functions x and w, *convolution* $w * x$ is defined as

$$(w * x)(t) = \int x(t-a)w(a)\,\mathrm{d}a.$$

For vectors, we have

lze řešit pomocí FFT

$$(\boldsymbol{w} * \boldsymbol{x})_t = \sum\nolimits_i x_{t-i} w_i.$$



Convolution operation can be generalized to two dimensions by

$$(\boldsymbol{K} * \boldsymbol{I})_{i,j} = \sum\nolimits_{m,n} \underline{\boldsymbol{I}_{i-m,j-n}} \boldsymbol{K}_{m,n}.$$

my jsme vzali W^T

Closely related is *cross-correlation*, where K is flipped:

$$(\boldsymbol{K} \star \boldsymbol{I})_{i,j} = \sum\nolimits_{m,n} \boldsymbol{I}_{i+m,j+n} \boldsymbol{K}_{m,n}.$$

Convolution Layer

The K is usually called a **kernel** or a **filter**.

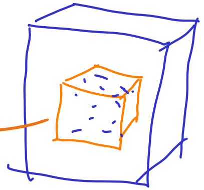
Note that usually we have a whole vector of values for a single pixel, the so-called **channels**. These single pixel channel values have no longer any spacial structure, so the kernel contains a different set of weights for every input dimension, obtaining

$$(\mathsf{K} \star \mathsf{I})_{i,j} = \sum_{m,n,c} \mathsf{I}_{i+m,j+n,c} \mathsf{K}_{m,n,c}.$$

→ kanál

≈ jako třeba RGB

kernel



Furthermore, we usually want to be able to specify the output dimensionality similarly to for example a fully connected layer – the number of **output channels** for every pixel. Each output channel is then the output of an independent convolution operation, so we can consider K to be a four-dimensional tensor and the convolution if computed as

→ můžu si vybrat, kolik chci výstupních kanálů

Kdybych měl kernel 1x1, tak je to jako na fully connected, akorát je to sdílené pro všechny featury

$$(\mathsf{K} \star \mathsf{I})_{i,j,o} = \sum_{m,n,c} \mathsf{I}_{i+m,j+n,c} \mathsf{K}_{m,n,c,o}.$$

pro každý to dělám zvlášť

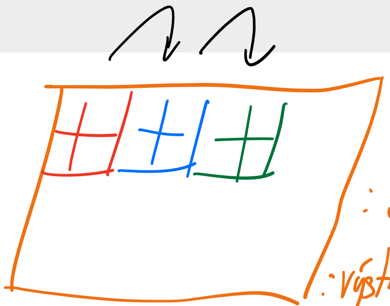
takže z toho můžu udělat 4D tensor

→ každý výstupní kanál má vlastní kernel

Convolution Layer

To arrive at the complete convolution layer, we need to specify:

- the width W and height H of the kernel;
- the number of output channels F; → takhle bude posouvání:
- the **stride** denoting that every output pixel is computed for every **stride**-th input pixel (e.g., the output is half the size if stride is 2).



: stride = 2

: výstup je poloviční

Considering an input image with C channels, the convolution layer is then parametrized by a kernel K of total size $W \times H \times C \times F$ and is computed as

$$(\mathsf{K} \star \mathsf{I})_{i,j,o} = \sum_{m,n,c} \mathsf{I}_{i \cdot S+m, j \cdot S+n, c} \mathsf{K}_{m,n,c,o}.$$

Note that while only local interactions are performed in the image spacial dimensions (width and height), we combine input channels in a fully connected manner.

Uvažme, že $i,j = 1,1$ $\to$ pak mám obrázek o velikosti jednoho pixelu.

Pak $W_{i,j,c,o}$ bude matice $W_{c,o}$, která bude mít váhu pro každou dvojici vstupního a výstupního kanálu. A to je najednou přesně to, co děláme v dense vrstvě.

- akorát to běží přes všechny pozice toho obrázku sdíleně

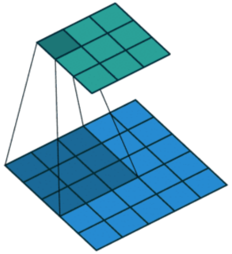
There are multiple padding schemes, most common are:

- `valid`: Only use valid pixels, which causes the result to be smaller than the input.
- `same`: Pad original image with zero pixels so that the result is exactly the size of the input.

Illustration of the padding schemes and different strides for a 3×3 kernel:

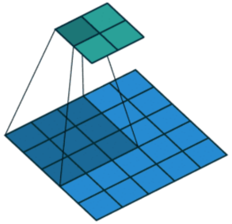
dochází k ořezu

- **valid** padding, stride=1:



https://github.com/vdumoulin/conv_arithmetic

stride=2:

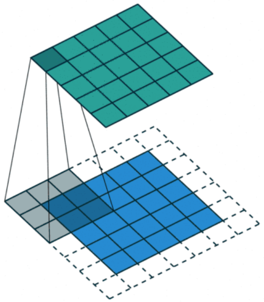


https://github.com/vdumoulin/conv_arithmetic

když mám těch vrstev hodně, tak to potřebuju, abych neztrácel moc dat

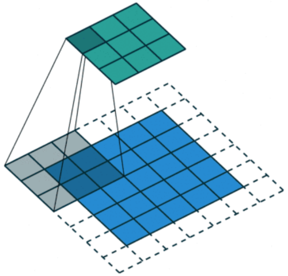
- **same** padding, stride=1:

vyplním 0



https://github.com/vdumoulin/conv_arithmetic

stride=2:



https://github.com/vdumoulin/conv_arithmetic

Convolution Layer Representation

There are two prevalent image formats (called `data_format` in Keras):

- `channels_last`: The dimensions of the 4-dimensional image tensor are batch, height, width, and channels.

  The original TensorFlow and Keras format, faster on CPU.

- `channels_first`: The dimensions of the 4-dimensional image tensor are batch, channel, height, and width.

  Originally faster on GPUs, nowadays `channels_last` is faster on newer GPUs; used by PyTorch format.

In TensorFlow, data is represented using the `channels_last` approach and the runtime will automatically convert it to `channels_first` if it is more suitable for available hardware (especially for a GPU).

In PyTorch, you can decide which memory representation you want, with the shape formally being always `channels_first`.

Pooling

Pooling is an operation similar to convolution, but we perform a fixed operation instead of multiplying by a kernel.

- Max pooling (minor translation invariance)
- Average pooling

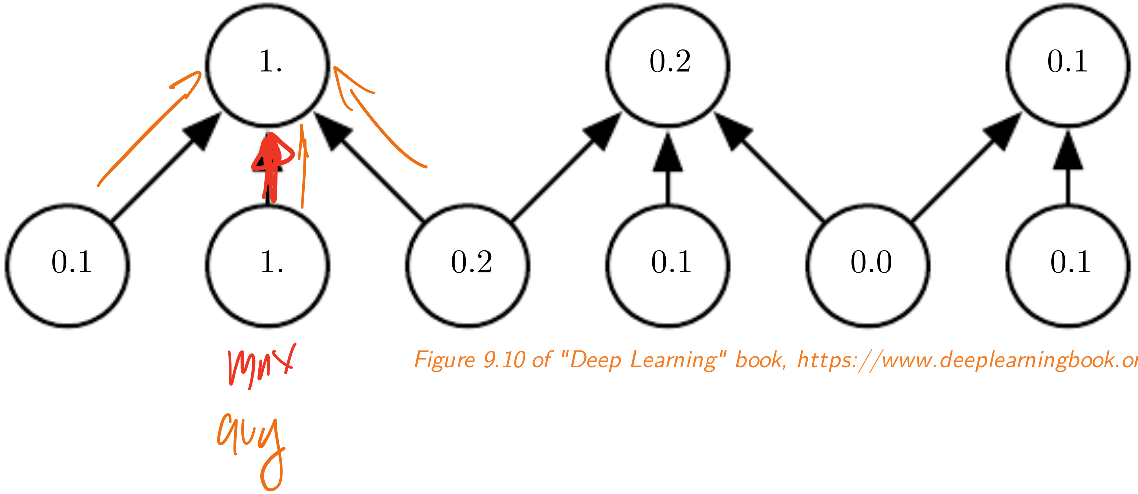


Figure 9.10 of "Deep Learning" book, https://www.deeplearningbook.org

High-level CNN Architecture

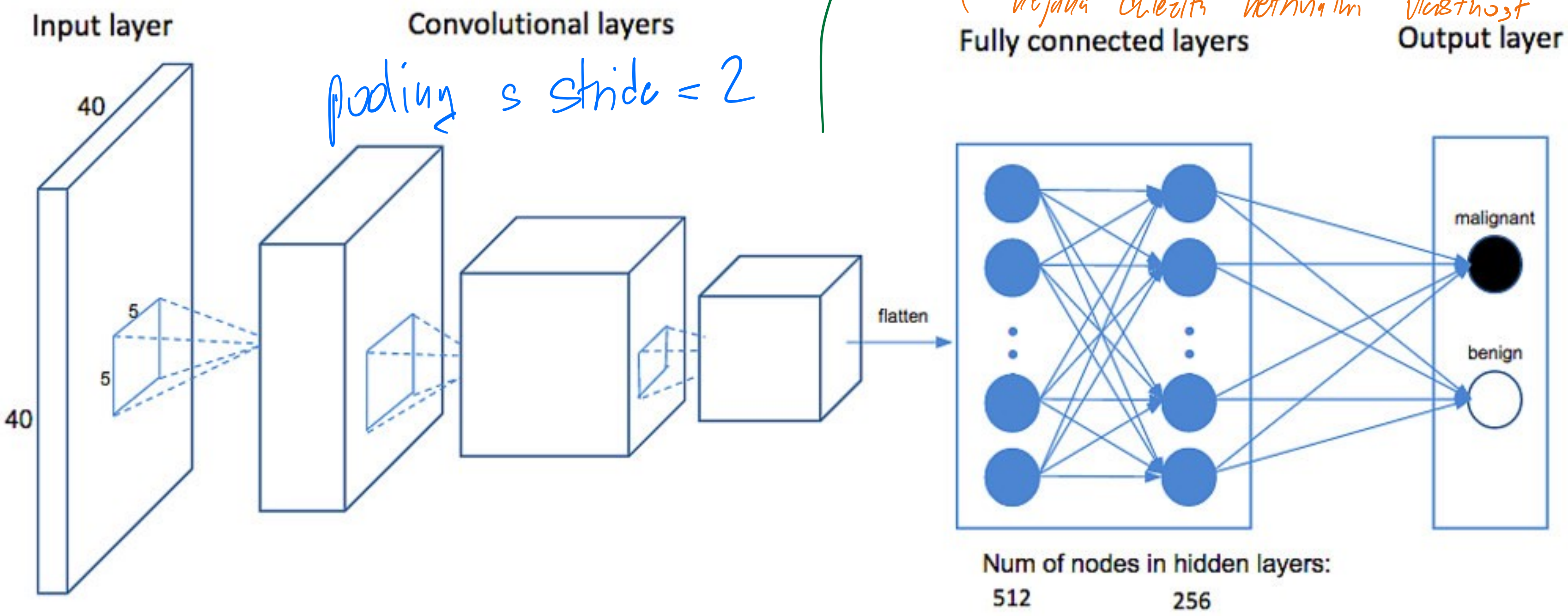
We repeatedly use the following block:

1. Convolution operation
2. Non-linear activation (usually ReLU)
3. Pooling

zmenšuju počet regionů, které interpretuju

tady už feature není pixel, jako spíš nějaká důležitá netriviální vlastnost

pooling s stride = 2



*https://cdn-images-1.medium.com/max/1200/0*QyXSpqpm1wc_Dt6V.*

AlexNet – 2012 (16.4% ILSVRC top-5 error)

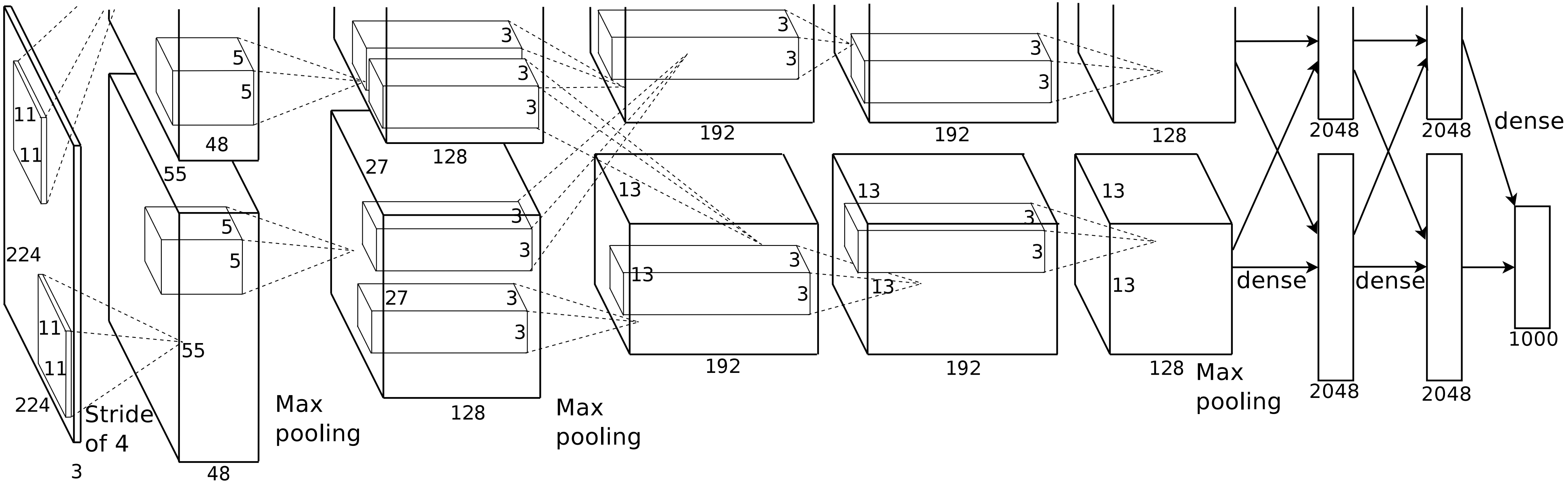


Figure 2: An illustration of the architecture of our CNN, explicitly showing the delineation of responsibilities between the two GPUs. One GPU runs the layer-parts at the top of the figure while the other runs the layer-parts at the bottom. The GPUs communicate only at certain layers. The network's input is 150,528-dimensional, and the number of neurons in the network's remaining layers is given by 253,440–186,624–64,896–64,896–43,264–4096–4096–1000.

Figure 2 of "ImageNet Classification with Deep Convolutional Neural Networks" by Alex Krizhevsky et al.

AlexNet – 2012 (16.4% ILSVRC top-5 error)

Training details:

- 61M parameters, 2 GPUs for 5-6 days
- SGD with batch size 128, momentum 0.9, L^2 regularization strength (weight decay) 0.0005
  - $\boldsymbol{v} \leftarrow 0.9 \cdot \boldsymbol{v} - \alpha \cdot \frac{\partial L}{\partial \boldsymbol{\theta}} - 0.0005 \cdot \alpha \cdot \boldsymbol{\theta}$
  - $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \boldsymbol{v}$
- initial learning rate 0.01, manually divided by 10 when validation error rate stopped improving
- ReLU nonlinearities
- dropout with rate 0.5 on the fully-connected layers (except for the output layer)
- data augmentation using translations and horizontal reflections (choosing random 224×224 patches from 256×256 images)
  - during inference, 10 patches are used (four corner patches and a center patch, as well as their reflections)

AlexNet – ReLU vs tanh

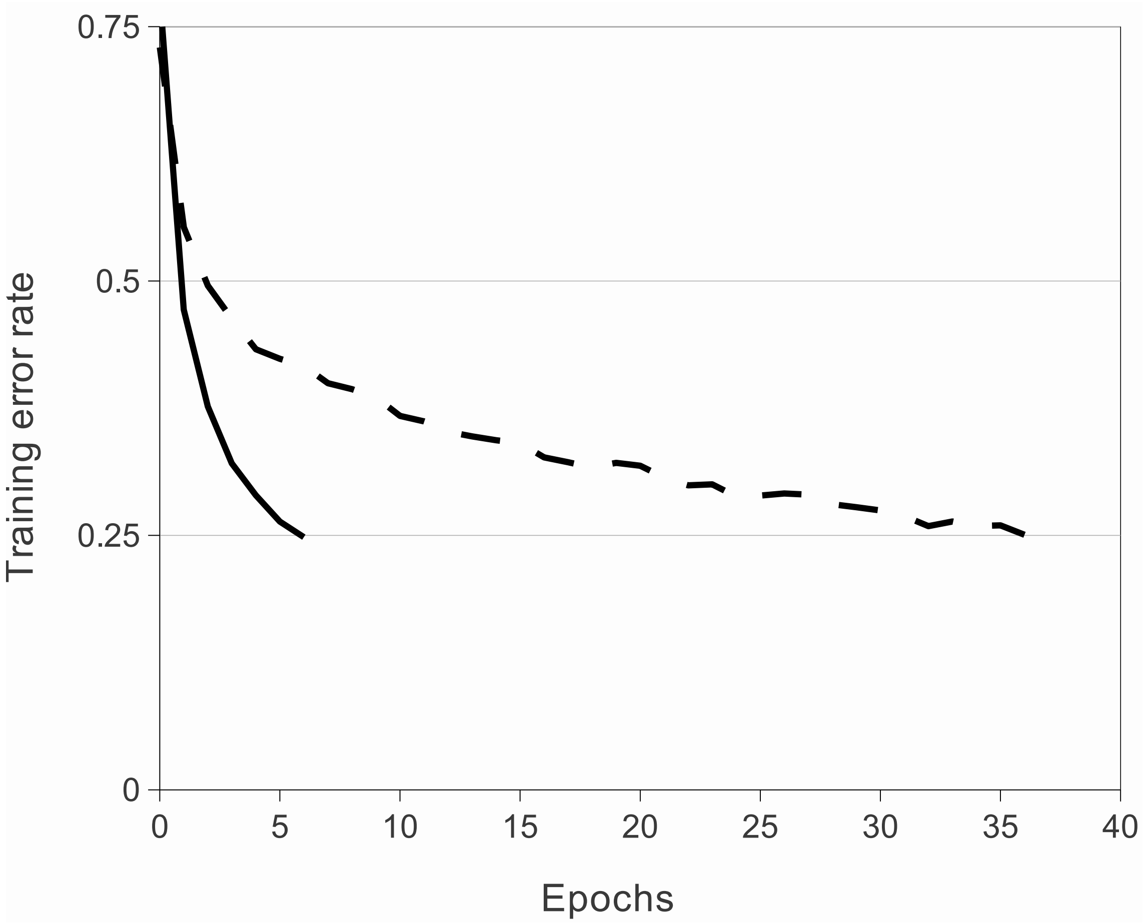


Figure 1 of "ImageNet Classification with Deep Convolutional Neural Networks" by Alex Krizhevsky et al.

AlexNet built on already existing CNN architectures, mostly on LeNet, which achieved 0.8% test error on MNIST.

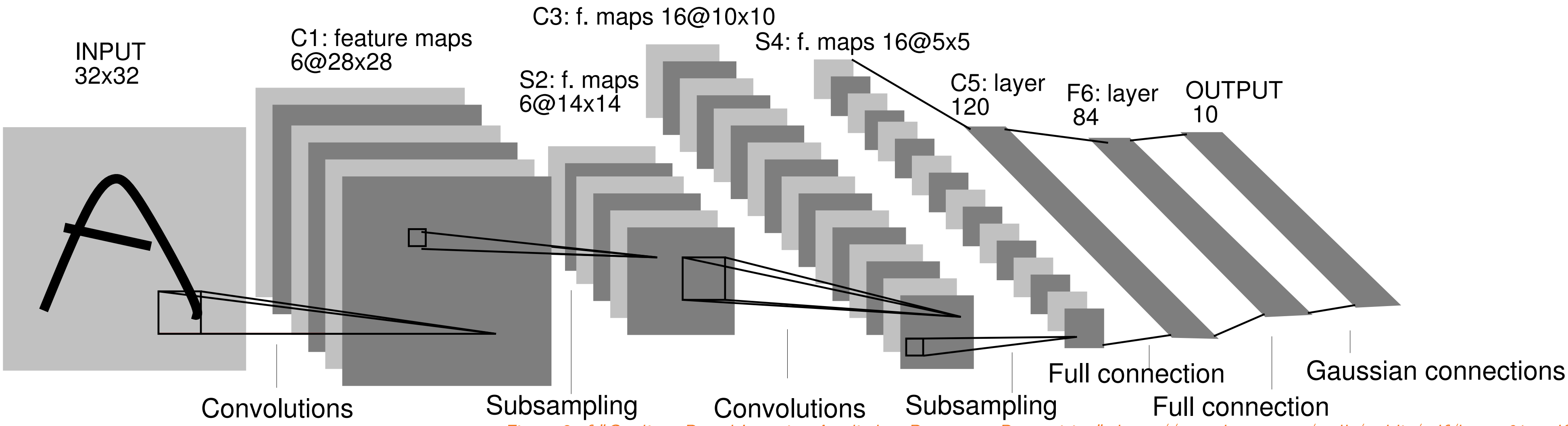


Figure 2 of "Gradient-Based Learning Applied to Document Recognition", http://yann.lecun.com/exdb/publis/pdf/lecun-01a.pdf

Similarities in Primary Visual Cortex (V1) and CNNs

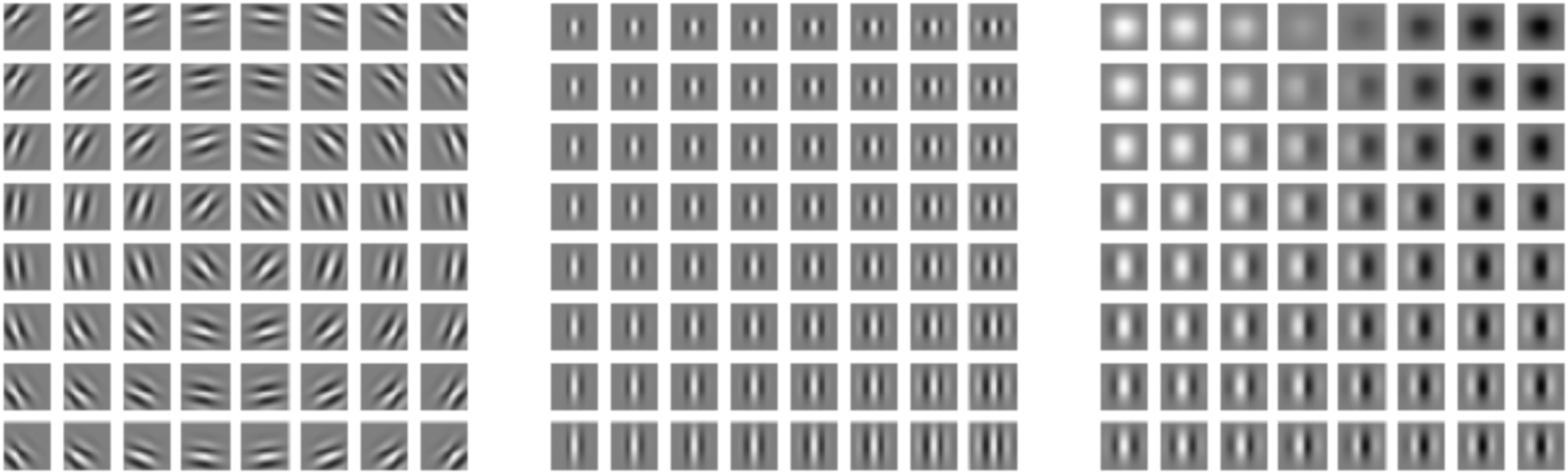
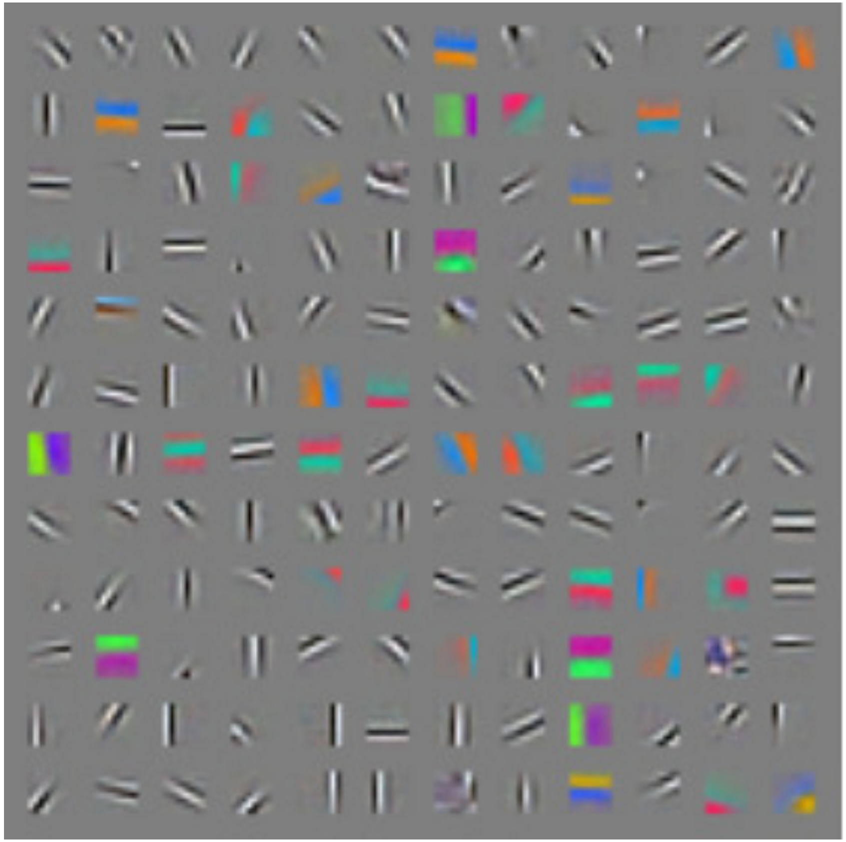


Figure 9.18 of "Deep Learning" book, https://www.deeplearningbook.org

The primary visual cortex recognizes Gabor functions.



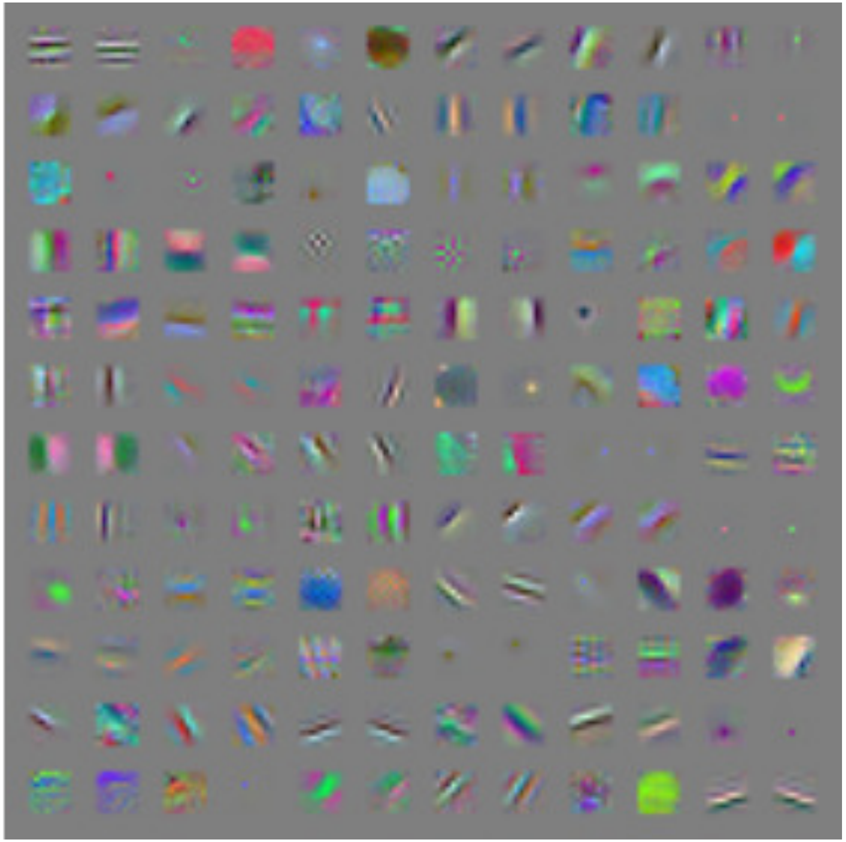


Figure 9.19 of "Deep Learning" book, https://www.deeplearningbook.org

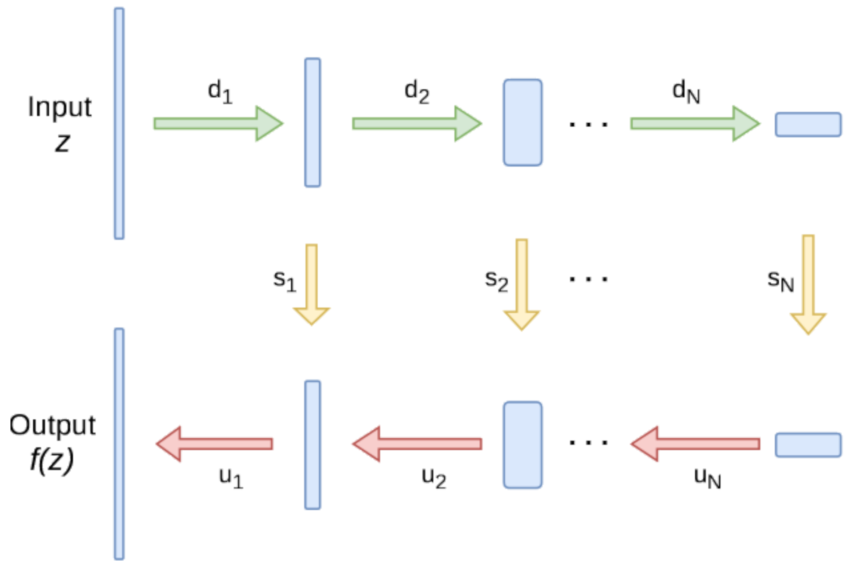
Similar functions are recognized in the first layer of a CNN.

CNNs as Regularizers – Deep Prior


(a) Ground truth
(b) SRResNet [18], **Trained**
(c) Bicubic, **Not trained**
(d) Deep prior, **Not trained**

Figure 1 of "Deep Image Prior", https://arxiv.org/abs/1711.10925

CNNs as Regularizers – Deep Prior



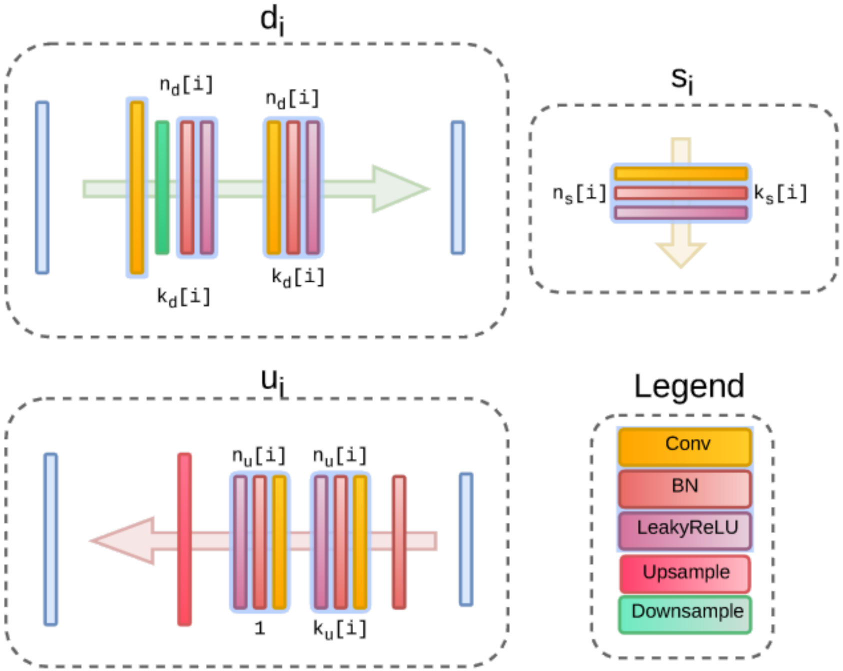


Figure 1 of "Deep Image Prior" supplementary materials, https://arxiv.org/abs/1711.10925

Random noise from $U[0, \frac{1}{10}]$ used on input; in large inpainting, meshgrid is used instead and the skip-connections are not used.

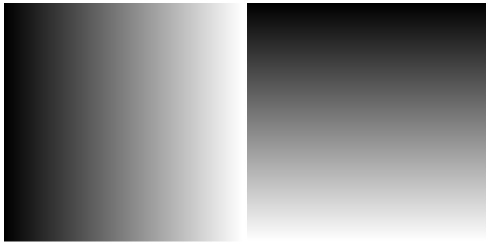


Figure 2 of "Deep Image Prior" supplementary materials, https://arxiv.org/abs/1711.10925

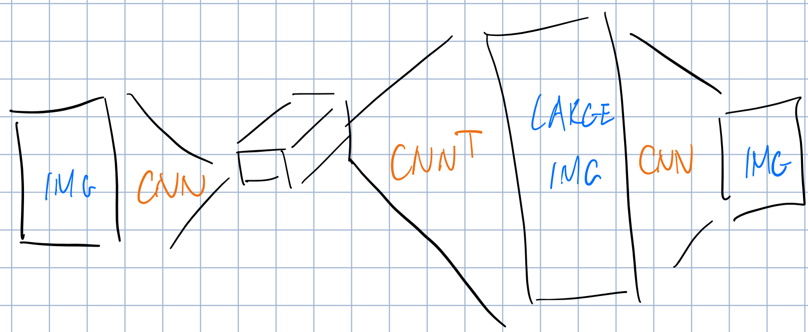
CNNs as Regularizers – Deep Prior

(a) Original image (b) Corrupted image (c) Shepard networks [26] (d) Deep Image Prior

(e) Original image (f) Corrupted image (g) [24], PSNR = 28.1 (h) Deep Img. Prior, PSNR = 30.9

Figure 7 of "Deep Image Prior", https://arxiv.org/abs/1711.10925v2

Tady se síť na obrázcích s textem učila znovu ze šumu vytvořit ten obrázek.



Ta síť pak vždy nejdřív vygenerovala obrázek bez textu/šumu a až pak se horko-těžko snažila vytvořit šum. Je to proto, že ten šum je příliš lokálního a unikátního charakteru.

→ tady síť měla doplnit okolí – a doplnila sama o sobě přesně to, co jsme chtěli.

Figure 5: **Inpainting diversity.** Left: original image (black pixels indicate holes). The remaining four images show results obtained using deep prior corresponding to different input vector z.

Figure 5 of "Deep Image Prior" supplementary materials, https://arxiv.org/abs/1711.10925

[Deep Prior paper website with supplementary material]()

VGG – 2014 (6.8% ILSVRC top-5 error)

ConvNet Configuration					
A	A-LRN	B	C	D	E
11 weight layers	11 weight layers	13 weight layers	16 weight layers	16 weight layers	19 weight layers
input (224 × 224 RGB image)					
conv3-64	conv3-64 **LRN**	conv3-64 **conv3-64**	conv3-64 conv3-64	conv3-64 conv3-64	conv3-64 conv3-64
maxpool					
conv3-128	conv3-128	conv3-128 **conv3-128**	conv3-128 conv3-128	conv3-128 conv3-128	conv3-128 conv3-128
maxpool					
conv3-256 conv3-256	conv3-256 conv3-256	conv3-256 conv3-256	conv3-256 conv3-256 **conv1-256**	conv3-256 conv3-256 **conv3-256**	conv3-256 conv3-256 conv3-256 **conv3-256**
maxpool					
conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512 **conv1-512**	conv3-512 conv3-512 **conv3-512**	conv3-512 conv3-512 conv3-512 **conv3-512**
maxpool					
conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512 **conv1-512**	conv3-512 conv3-512 **conv3-512**	conv3-512 conv3-512 conv3-512 **conv3-512**
maxpool					
FC-4096					
FC-4096					
FC-1000					
soft-max					

Figure 1 of "Very Deep Convolutional Networks For Large-Scale Image Recognition", https://arxiv.org/abs/1409.1556

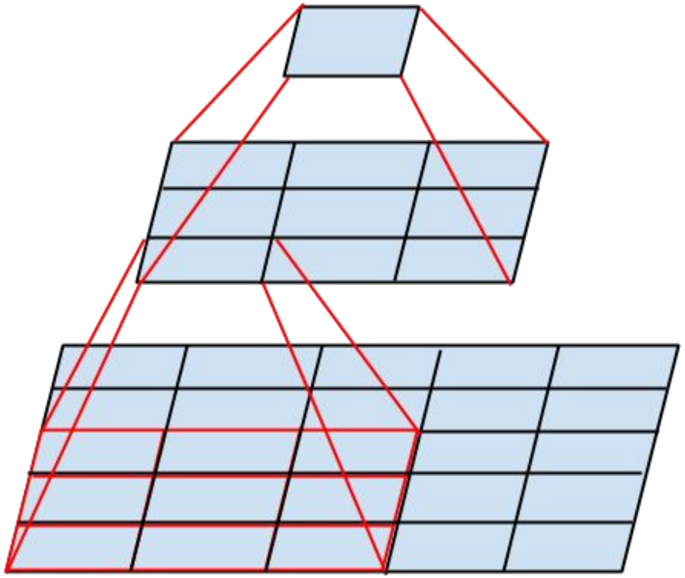


Figure 1 of "Rethinking the Inception Architecture for Computer Vision", https://arxiv.org/abs/1512.00567

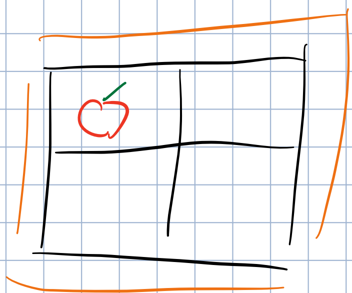
Table 2: **Number of parameters** (in millions).

Network	A,A-LRN	B	C	D	E
Number of parameters	133	133	134	138	144

Figure 2 of "Very Deep Convolutional Networks For Large-Scale Image Recognition", https://arxiv.org/abs/1409.1556

Proč samý MAXPOOLING?

Protože když mám nějaký k podregiony, ze kterých sbírám features, tak mi zajímá, ze kterých směrů jde ta informace nejvíc, nikoliv jak moc je obecně zastoupená.



→ tady určitě je jablko, přestože v průměru tam moc není...

VGG – 2014 (6.8% ILSVRC top-5 error)

Training detail similar to AlexNet:

- SGD with batch size ~~128~~ 256, momentum 0.9, weight decay 0.0005
- initial learning rate 0.01, manually divided by 10 when validation error rate stopped improving
- ReLU nonlinearities
- dropout with rate 0.5 on the fully-connected layers (except for the output layer)
- data augmentation using translations and horizontal reflections (choosing random 224×224 patches from 256×256 images)
    - additionally, a multi-scale training and evaluation was performed. During training, each image was resized so that its smaller size was equal to S, which was sampled uniformly from $[256, 512]$ – kernel už není imunní vůči velikosti objektu, takže je potřeba obrázky velikostně augmentovat
    - during test time, the image was rescaled three times so that the smaller size was $256, 384, 512$, respectively, and the results on the three images were averaged

VGG – 2014 (6.8% ILSVRC top-5 error)

Table 3: **ConvNet performance at a single test scale.**

ConvNet config. (Table 1)	smallest image side		top-1 val. error (%)	top-5 val. error (%)
	train (S)	test (Q)		
A	256	256	29.6	10.4
A-LRN	256	256	29.7	10.5
B	256	256	28.7	9.9
C	256	256	28.1	9.4
	384	384	28.1	9.3
	[256;512]	384	27.3	8.8
D	256	256	27.0	8.8
	384	384	26.8	8.7
	[256;512]	384	25.6	8.1
E	256	256	27.3	9.0
	384	384	26.9	8.7
	[256;512]	384	**25.5**	**8.0**

Table 3 of "Very Deep Convolutional Networks For Large-Scale Image Recognition", https://arxiv.org/abs/1409.1556

Table 4: **ConvNet performance at multiple test scales.**

ConvNet config. (Table 1)	smallest image side		top-1 val. error (%)	top-5 val. error (%)
	train (S)	test (Q)		
B	256	224,256,288	28.2	9.6
C	256	224,256,288	27.7	9.2
	384	352,384,416	27.8	9.2
	$[256; 512]$	256,384,512	26.3	8.2
D	256	224,256,288	26.6	8.6
	384	352,384,416	26.5	8.6
	$[256; 512]$	256,384,512	**24.8**	**7.5**
E	256	224,256,288	26.9	8.7
	384	352,384,416	26.7	8.6
	$[256; 512]$	256,384,512	**24.8**	**7.5**

Table 4 of "Very Deep Convolutional Networks For Large-Scale Image Recognition", https://arxiv.org/abs/1409.1556

VGG – 2014 (6.8% ILSVRC top-5 error)

Method	top-1 val. error (%)	top-5 val. error (%)	top-5 test error (%)
VGG (2 nets, multi-crop & dense eval.)	**23.7**	**6.8**	**6.8**
VGG (1 net, multi-crop & dense eval.)	24.4	7.1	7.0
VGG (ILSVRC submission, 7 nets, dense eval.)	24.7	7.5	7.3
GoogLeNet (Szegedy et al., 2014) (1 net)	-	7.9	
GoogLeNet (Szegedy et al., 2014) (7 nets)	-	**6.7**	
MSRA (He et al., 2014) (11 nets)	-	-	8.1
MSRA (He et al., 2014) (1 net)	27.9	9.1	9.1
Clarifai (Russakovsky et al., 2014) (multiple nets)	-	-	11.7
Clarifai (Russakovsky et al., 2014) (1 net)	-	-	12.5
Zeiler & Fergus (Zeiler & Fergus, 2013) (6 nets)	36.0	14.7	14.8
Zeiler & Fergus (Zeiler & Fergus, 2013) (1 net)	37.5	16.0	16.1
OverFeat (Sermanet et al., 2014) (7 nets)	34.0	13.2	13.6
OverFeat (Sermanet et al., 2014) (1 net)	35.7	14.2	-
Krizhevsky et al. (Krizhevsky et al., 2012) (5 nets)	38.1	16.4	16.4
Krizhevsky et al. (Krizhevsky et al., 2012) (1 net)	40.7	18.2	-

Figure 2 of "Very Deep Convolutional Networks For Large-Scale Image Recognition", https://arxiv.org/abs/1409.1556

Inception block:

tyhle jsou hrozně drahý

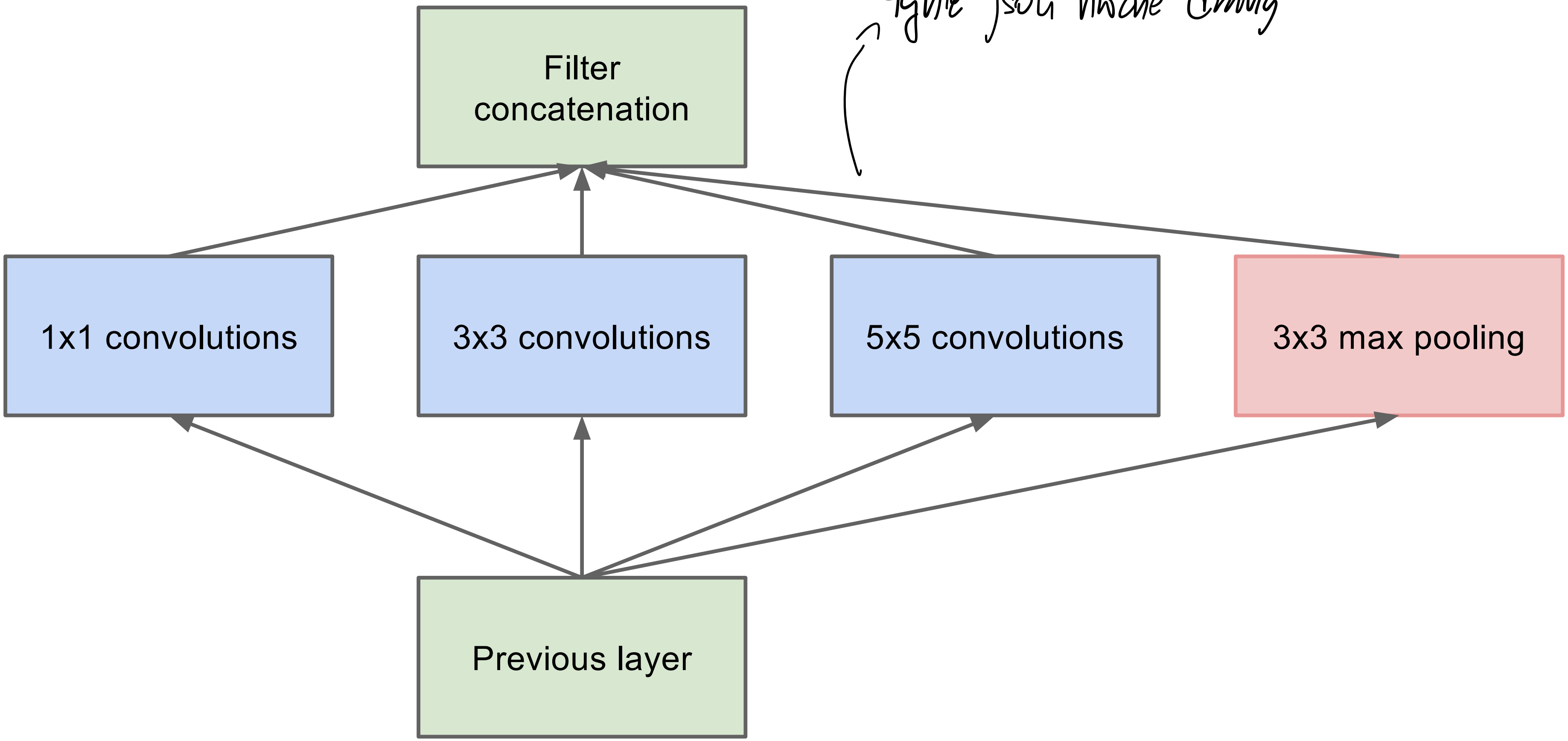


Figure 2 of "Going Deeper with Convolutions", https://arxiv.org/abs/1409.4842

Inception block with dimensionality reduction:

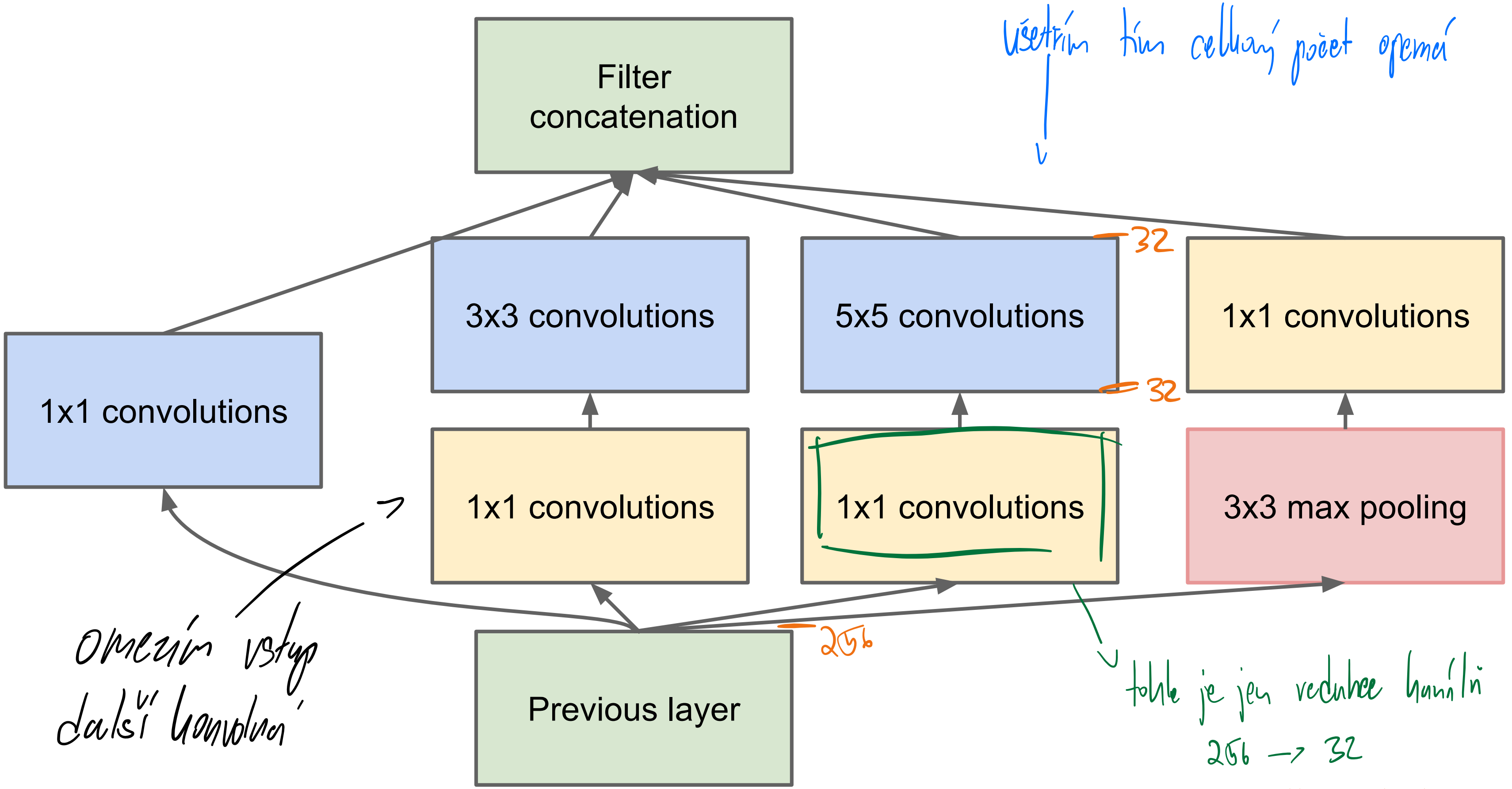


Figure 2 of "Going Deeper with Convolutions", https://arxiv.org/abs/1409.4842

Inception (GoogLeNet) – 2014 (6.7% ILSVRC top-5 error)

type	patch size/ stride	output size	depth	#1×1	#3×3 reduce	#3×3	#5×5 reduce	#5×5	pool proj	params	ops
convolution	7×7/2	112×112×64	1							2.7K	34M
max pool	3×3/2	56×56×64	0								
convolution	3×3/1	56×56×192	2		64	192				112K	360M
max pool	3×3/2	28×28×192	0								
inception (3a)		28×28×256	2	64	96	128	16	32	32	159K	128M
inception (3b)		28×28×480	2	128	128	192	32	96	64	380K	304M
max pool	3×3/2	14×14×480	0								
inception (4a)		14×14×512	2	192	96	208	16	48	64	364K	73M
inception (4b)		14×14×512	2	160	112	224	24	64	64	437K	88M
inception (4c)		14×14×512	2	128	128	256	24	64	64	463K	100M
inception (4d)		14×14×528	2	112	144	288	32	64	64	580K	119M
inception (4e)		14×14×832	2	256	160	320	32	128	128	840K	170M
max pool	3×3/2	7×7×832	0								
inception (5a)		7×7×832	2	256	160	320	32	128	128	1072K	54M
inception (5b)		7×7×1024	2	384	192	384	48	128	128	1388K	71M
avg pool	7×7/1	1×1×1024	0								
dropout (40%)		1×1×1024	0								
linear		1×1×1000	1							1000K	1M
softmax		1×1×1000	0								

→ avg. pooling přes celý obrázek můžu dělat pouze pokud mám dost hlubokou síť.

Table 1 of "Going Deeper with Convolutions", https://arxiv.org/abs/1409.4842

Inception (GoogLeNet) – 2014 (6.7% ILSVRC top-5 error)

pak už jsem
tohle zahotvený
a nepotřebuju
k zahotvení
dense vrstvu

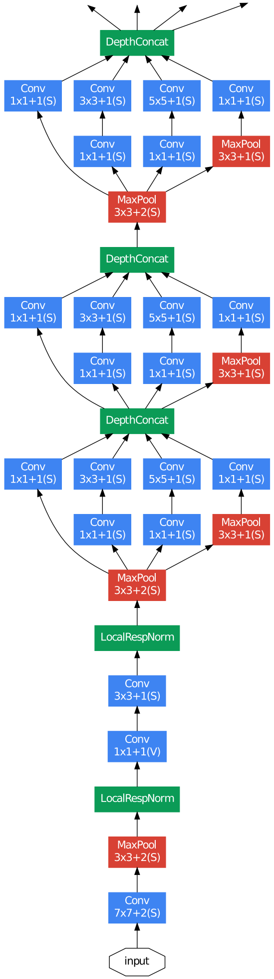


Figure 3 of "Going Deeper with Convolutions", https://arxiv.org/abs/1409.4842

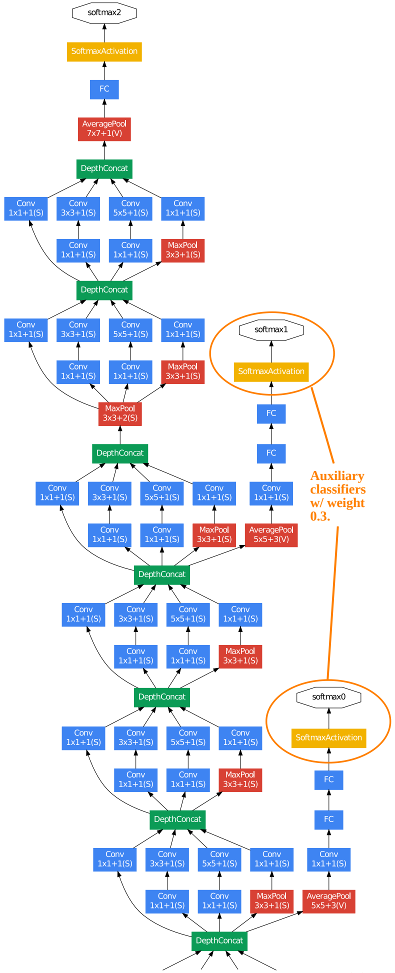


Figure 3 of "Going Deeper with Convolutions", https://arxiv.org/abs/1409.4842

Training details:

- SGD with momentum 0.9
- fixed learning rate schedule of decreasing the learning rate by 4% each 8 epochs
- during test time, the image was rescaled four times so that the smaller size was $256, 288, 320, 352$, respectively.

  For each image, the left, center and right square was considered, and from each square six crops of size 224×224 were extracted (4 corners, middle crop and the whole scaled-down square) together with their horizontal flips, arriving at $4 \cdot 3 \cdot 6 \cdot 2 = 144$ crops per image
- 7 independently trained models were ensembled

Number of models	Number of Crops	Cost	Top-5 error	compared to base
1	1	1	10.07%	base
1	10	10	9.15%	-0.92%
1	144	144	7.89%	-2.18%
7	1	7	8.09%	-1.98%
7	10	70	7.62%	-2.45%
7	144	1008	6.67%	-3.45%

Table 3 of "Going Deeper with Convolutions", https://arxiv.org/abs/1409.4842

Batch Normalization

Internal covariate shift refers to the change in the distributions of hidden node activations due to the updates of network parameters during training.

Let $\boldsymbol{x} = (x_1, \ldots, x_d)$ be d-dimensional input. We would like to normalize each dimension as

výstupy vrstvy

$$\hat{x}_i = \frac{x_i - \mathbb{E}[x_i]}{\sqrt{\operatorname{Var}[x_i]}}.$$

budou normální, platí že $\mathbb{E}[\hat{x}] = 0$, $\operatorname{Var}(\hat{x}) = 1$

Furthermore, it may be advantageous to learn suitable scale γ_i and shift β_i to produce normalized value

$$y_i = \gamma_i \hat{x}_i + \beta_i. \quad \longrightarrow \quad \mathbb{E}[y_i] = \beta_i, \quad \operatorname{Var}(y_i) = \gamma_i^2$$

a pak to naškáluju

Batch Normalization

Batch normalization of a mini-batch of m examples $(\boldsymbol{x}^{(1)}, \ldots, \boldsymbol{x}^{(m)})$ is the following:

Inputs: Mini-batch $(\boldsymbol{x}^{(1)}, \ldots, \boldsymbol{x}^{(m)})$, $\varepsilon \in \mathbb{R}$ with default value 0.001
Parameters: $\boldsymbol{\beta}$ initialized to $\mathbf{0}$, $\boldsymbol{\gamma}$ initialized to $\mathbf{1}$; both trained by the optimizer
Outputs: Normalized batch $(\boldsymbol{y}^{(1)}, \ldots, \boldsymbol{y}^{(m)})$

- $\boldsymbol{\mu} \leftarrow \frac{1}{m} \sum_{i=1}^{m} \boldsymbol{x}^{(i)}$
- $\boldsymbol{\sigma}^2 \leftarrow \frac{1}{m} \sum_{i=1}^{m} (\boldsymbol{x}^{(i)} - \boldsymbol{\mu})^2$

odhad na batchy

- $\hat{\boldsymbol{x}}^{(i)} \leftarrow (\boldsymbol{x}^{(i)} - \boldsymbol{\mu}) / \sqrt{\boldsymbol{\sigma}^2 + \varepsilon}$
- $\boldsymbol{y}^{(i)} \leftarrow \boldsymbol{\gamma} \odot \hat{\boldsymbol{x}}^{(i)} + \boxed{\boldsymbol{\beta}}$ → sem můžu přesunout bias

Batch normalization is added just before a nonlinearity f, and it is useless to add bias before it (because it will cancel out). Therefore, we replace $f(\boldsymbol{W}\boldsymbol{x} + \boldsymbol{b})$ by

$$f(BN(\boldsymbol{W}\boldsymbol{x})).$$

nejlepší je bias vyhodit, protože je zbytečný, ale dělá to pak bordel...

Batch Normalization during Inference

During inference, $\boldsymbol\mu$ and $\boldsymbol\sigma^2$ are fixed (so that prediction does not depend on other examples in a batch).

They could be precomputed after training on the whole training data, but in practice we estimate $\hat{\boldsymbol\mu}$ and $\hat{\boldsymbol\sigma}^2$ during training using an exponential moving average.

Additional Inputs: momentum $\tau \in \mathbb{R}$ with default value of 0.99

Additional Parameters: $\hat{\boldsymbol\mu}$ initialized to $\mathbf{0}$, $\hat{\boldsymbol\sigma}^2$ initialized to $\mathbf{1}$; both updated manually

During training, also perform:

- $\hat{\boldsymbol\mu} \leftarrow \tau\hat{\boldsymbol\mu} + (1-\tau)\boldsymbol\mu$
- $\hat{\boldsymbol\sigma}^2 \leftarrow \tau\hat{\boldsymbol\sigma}^2 + (1-\tau)\boldsymbol\sigma^2$

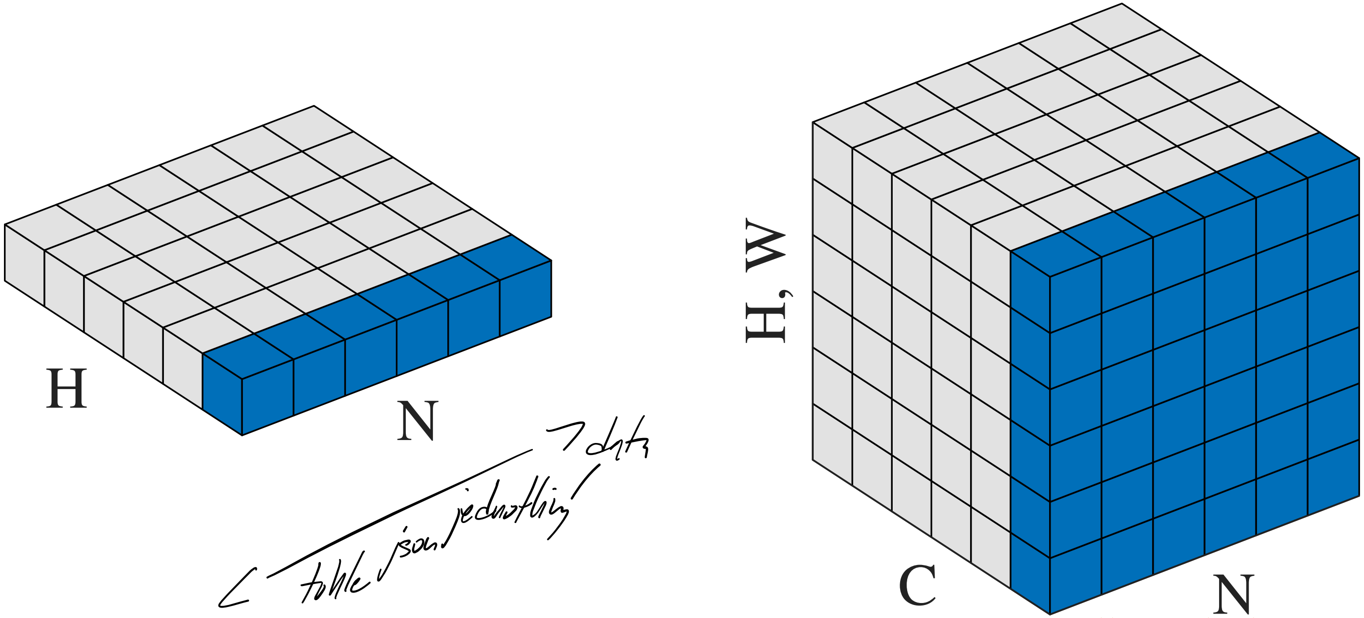
Batch normalization is then during inference computed as:

- $\hat{\boldsymbol x}^{(i)} \leftarrow (\boldsymbol x^{(i)} - \hat{\boldsymbol\mu})/\sqrt{\hat{\boldsymbol\sigma}^2 + \varepsilon}$
- $\boldsymbol y^{(i)} \leftarrow \boldsymbol\gamma \odot \hat{\boldsymbol x}^{(i)} + \boldsymbol\beta$

Batch Normalization

When a batch normalization is used on a fully connected layer, each neuron is normalized individually across the minibatch.

However, for convolutional networks we would like the normalization to honour their properties, most notably the shift invariance. We therefore normalize each channel across not only the minibatch, but also across all corresponding spacial/temporal locations.



← tohle jsou jednotlivé data

tady musím jít přes všechny příklady, všechny jejich pozice, v rámci jednoh. kanálu.

Adapted from Figure 2 of "Group Normalization", https://arxiv.org/abs/1803.08494

Takže mám tolik β, γ, kolik mám kanálů

Inception with BatchNorm (4.8% ILSVRC top-5 error)

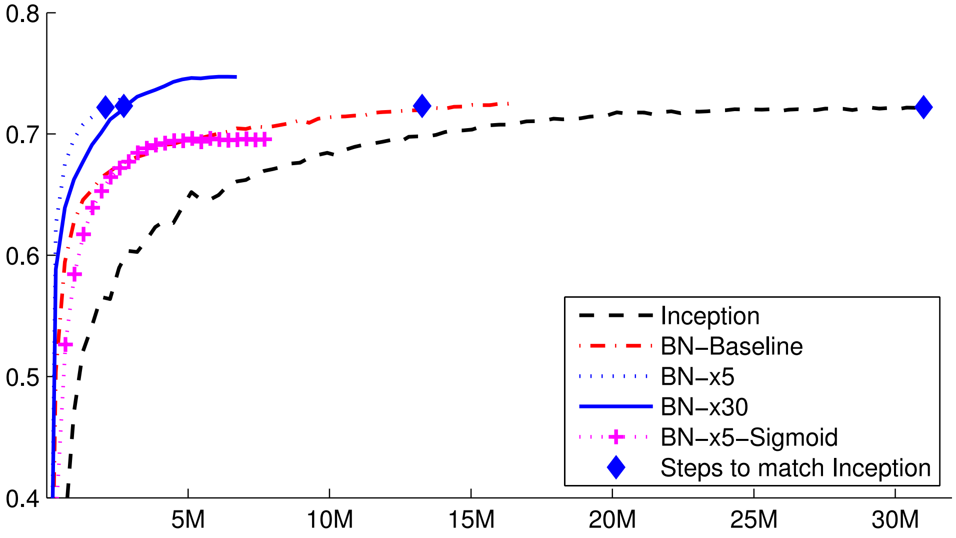


Figure 2: *Single crop validation accuracy of Inception and its batch-normalized variants, vs. the number of training steps.*

Model	Steps to 72.2%	Max accuracy
Inception	$31.0 \cdot 10^6$	72.2%
BN-Baseline	$13.3 \cdot 10^6$	72.7%
BN-x5	$2.1 \cdot 10^6$	73.0%
BN-x30	$2.7 \cdot 10^6$	74.8%
BN-x5-Sigmoid		69.8%

Figure 3: *For Inception and the batch-normalized variants, the number of training steps required to reach the maximum accuracy of Inception (72.2%), and the maximum accuracy achieved by the network.*

Figures 2 and 3 of "Batch Normalization: Accelerating Deep Network Training by Reducing Internal Covariate Shift", https://arxiv.org/abs/1502.03167

The BN-x5 and BN-x30 use 5/30 times larger initial learning rate, faster learning rate decay, no dropout, weight decay smaller by a factor of 5, and several more minor changes.

Inception v2 and v3 – 2015 (3.6% ILSVRC top-5 error)

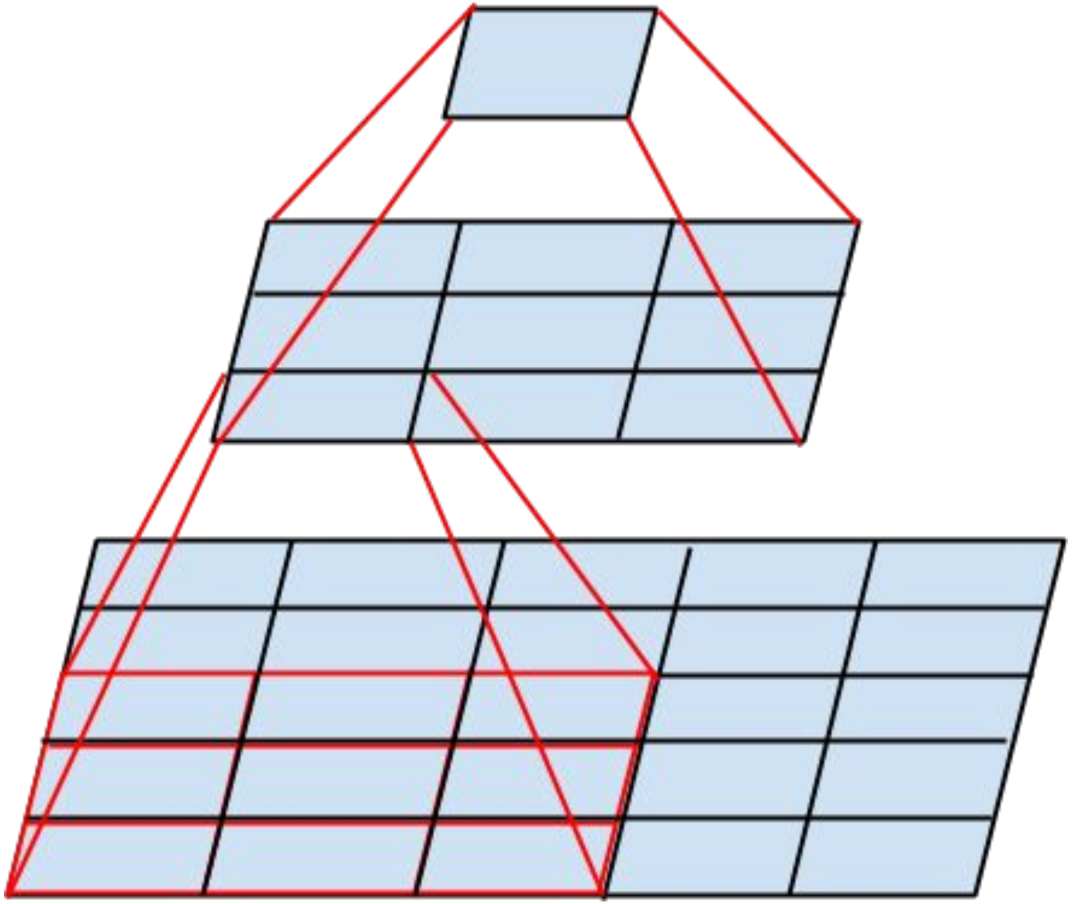


Figure 1 of "Rethinking the Inception Architecture for Computer Vision", https://arxiv.org/abs/1512.00567

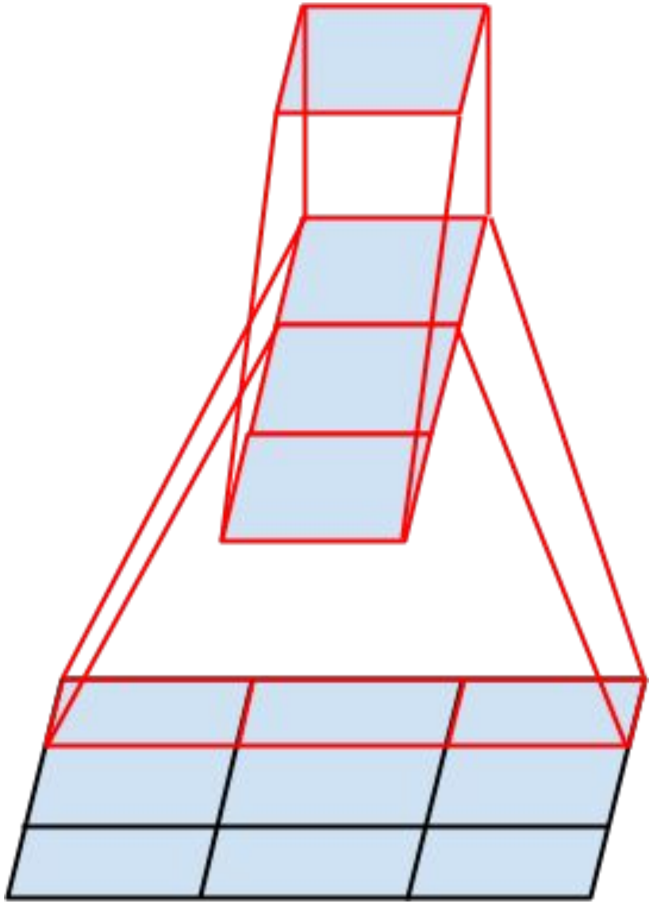


Figure 3 of "Rethinking the Inception Architecture for Computer Vision", https://arxiv.org/abs/1512.00567

Inception v2 and v3 – 2015 (3.6% ILSVRC top-5 error)

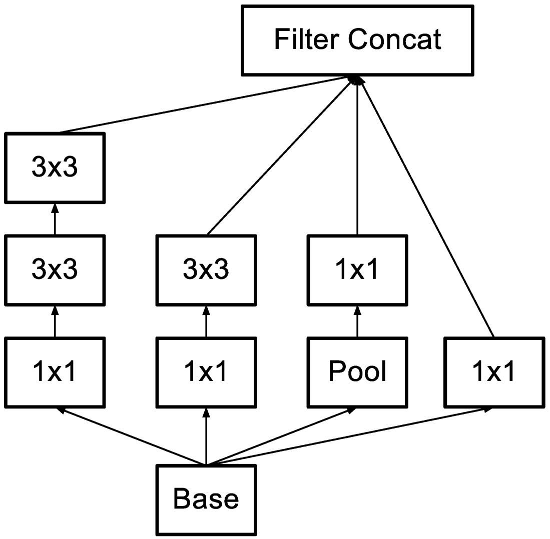


Figure 5. Inception modules where each 5 × 5 convolution is replaced by two 3 × 3 convolution, as suggested by principle 3 of Section 2.

Figure 5 of "Rethinking the Inception Architecture for Computer Vision", https://arxiv.org/abs/1512.00567

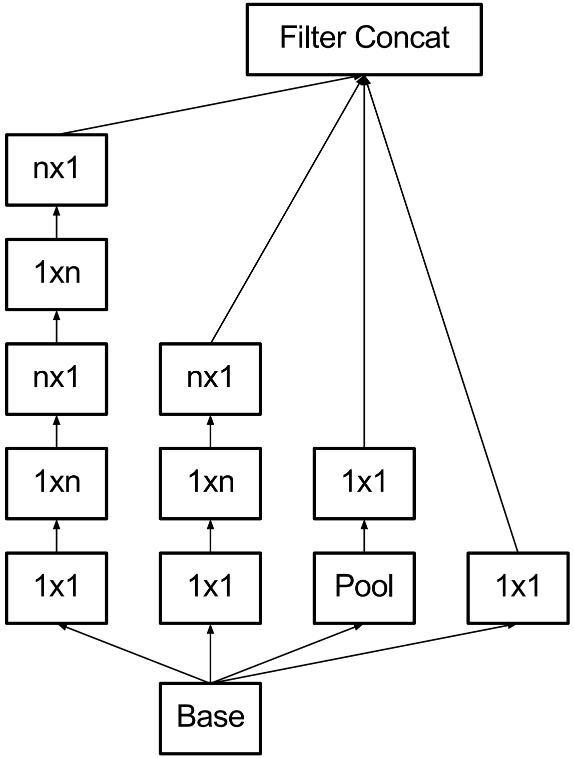


Figure 6. Inception modules after the factorization of the $n \times n$ convolutions. In our proposed architecture, we chose $n = 7$ for the 17×17 grid. (The filter sizes are picked using principle 3)

Figure 6 of "Rethinking the Inception Architecture for Computer Vision", https://arxiv.org/abs/1512.00567

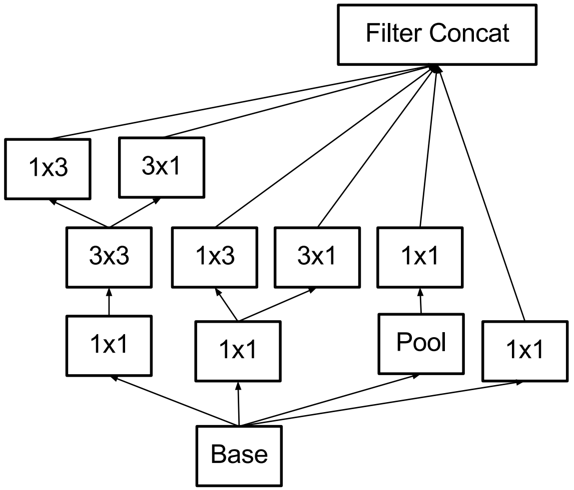


Figure 7. Inception modules with expanded the filter bank outputs. This architecture is used on the coarsest (8 × 8) grids to promote high dimensional representations, as suggested by principle 2 of Section 2. We are using this solution only on the coarsest grid, since that is the place where producing high dimensional sparse representation is the most critical as the ratio of local processing (by 1 × 1 convolutions) is increased compared to the spatial aggregation.

Figure 7 of "Rethinking the Inception Architecture for Computer Vision", https://arxiv.org/abs/1512.00567

Inception v2 and v3 – 2015 (3.6% ILSVRC top-5 error)

type	patch size/stride or remarks	input size
conv	$3\times3/2$	$299\times299\times3$
conv	$3\times3/1$	$149\times149\times32$
conv padded	$3\times3/1$	$147\times147\times32$
pool	$3\times3/2$	$147\times147\times64$
conv	$3\times3/1$	$73\times73\times64$
conv	$3\times3/2$	$71\times71\times80$
conv	$3\times3/1$	$35\times35\times192$
$3\times$Inception	As in figure 5	$35\times35\times288$
$5\times$Inception	As in figure 6	$17\times17\times768$
$2\times$Inception	As in figure 7	$8\times8\times1280$
pool	8×8	$8\times8\times2048$
linear	logits	$1\times1\times2048$
softmax	classifier	$1\times1\times1000$

Table 1 of "Rethinking the Inception Architecture for Computer Vision", https://arxiv.org/abs/1512.00567

Inception v2 and v3 – 2015 (3.6% ILSVRC top-5 error)

Training details:

- RMSProp with momentum of $\beta = 0.9$ and $\varepsilon = 1.0$
- batch size of 32 for 100 epochs
- initial learning rate of 0.045, decayed by 6% every two epochs
- gradient clipping with threshold 2.0 was used to stabilize the training
- label smoothing was first used in this paper, with $\alpha = 0.1$
- input image size enlarged to 299×299

Inception v2 and v3 – 2015 (3.6% ILSVRC top-5 error)

Network	**Top-1** Error	**Top-5** Error	**Cost** Bn Ops
GoogLeNet [20]	29%	9.2%	1.5
BN-GoogLeNet	26.8%	-	**1.5**
BN-Inception [7]	25.2%	7.8	2.0
Inception-v2	23.4%	-	3.8
Inception-v2 RMSProp	23.1%	6.3	3.8
Inception-v2 Label Smoothing	22.8%	6.1	3.8
Inception-v2 Factorized 7×7	21.6%	5.8	4.8
Inception-v2 BN-auxiliary	**21.2%**	**5.6%**	4.8

Table 3 of "Rethinking the Inception Architecture for Computer Vision", https://arxiv.org/abs/1512.00567

Inception v2 and v3 – 2015 (3.6% ILSVRC top-5 error)

Network	Crops Evaluated	Top-5 Error	Top-1 Error
GoogLeNet [20]	10	-	9.15%
GoogLeNet [20]	144	-	7.89%
VGG [18]	-	24.4%	6.8%
BN-Inception [7]	144	22%	5.82%
PReLU [6]	10	24.27%	7.38%
PReLU [6]	-	21.59%	5.71%
Inception-v3	12	19.47%	4.48%
Inception-v3	144	**18.77%**	**4.2%**

Table 4 of "Rethinking the Inception Architecture for Computer Vision", https://arxiv.org/abs/1512.00567

Network	Models Evaluated	Crops Evaluated	Top-1 Error	Top-5 Error
VGGNet [18]	2	-	23.7%	6.8%
GoogLeNet [20]	7	144	-	6.67%
PReLU [6]	-	-	-	4.94%
BN-Inception [7]	6	144	20.1%	4.9%
Inception-v3	4	144	**17.2%**	**3.58%***

Table 5 of "Rethinking the Inception Architecture for Computer Vision", https://arxiv.org/abs/1512.00567

ResNet – 2015 (3.6% ILSVRC top-5 error)

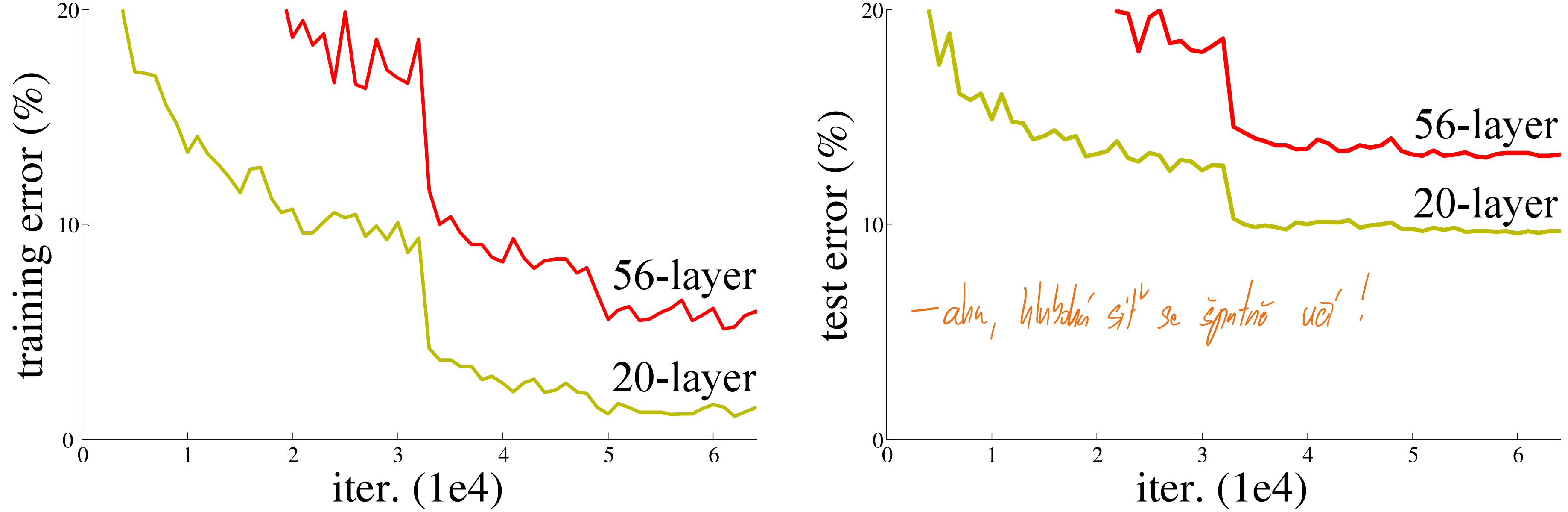


Figure 1. Training error (left) and test error (right) on CIFAR-10 with 20-layer and 56-layer "plain" networks. The deeper network has higher training error, and thus test error. Similar phenomena on ImageNet is presented in Fig. 4.

Figure 1 of "Deep Residual Learning for Image Recognition", https://arxiv.org/abs/1512.03385

„Co mám dělat jinak vůči vstupu?"

Implicitní kopíruje, přidám změnu

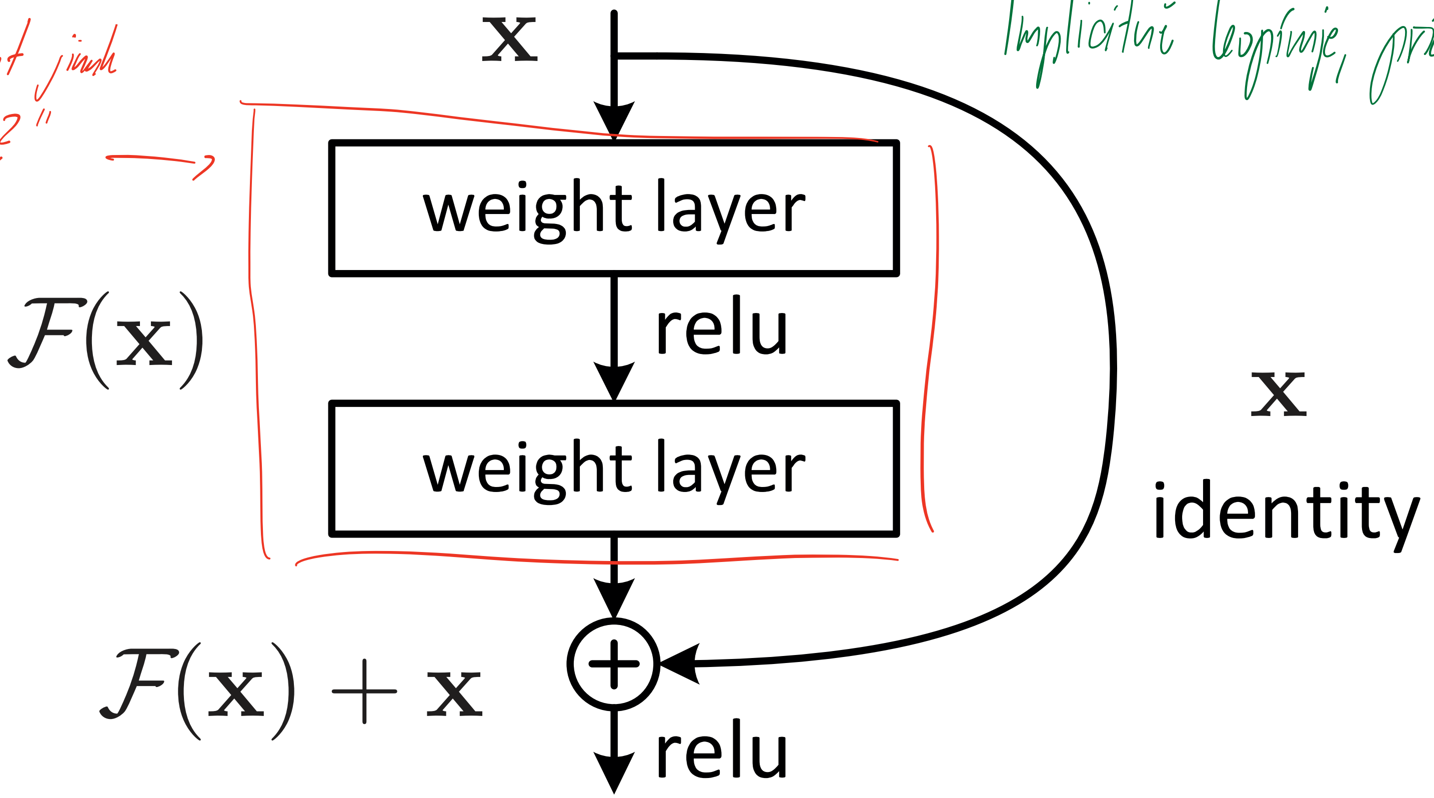


Figure 2. Residual learning: a building block.

Figure 2 of "Deep Residual Learning for Image Recognition", https://arxiv.org/abs/1512.03385

ResNet – 2015 (3.6% ILSVRC top-5 error)

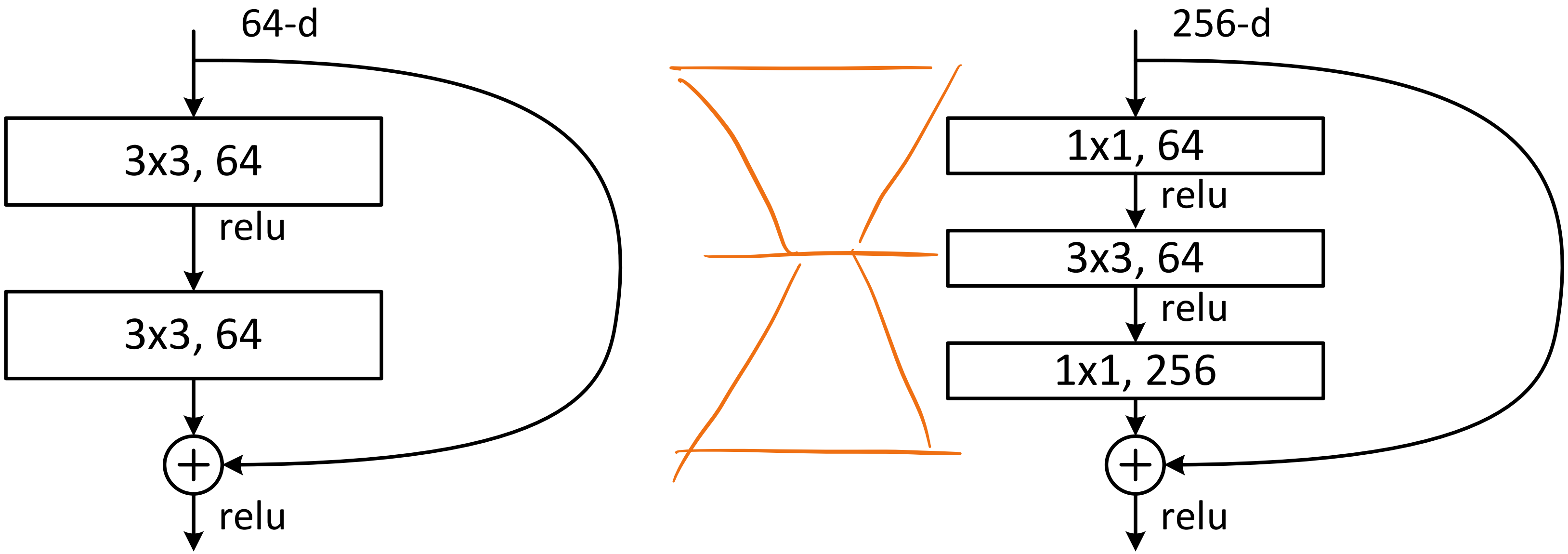


Figure 5. A deeper residual function $\mathcal{F}$ for ImageNet. Left: a building block (on 56×56 feature maps) as in Fig. 3 for ResNet-34. Right: a "bottleneck" building block for ResNet-50/101/152.

Figure 5 of "Deep Residual Learning for Image Recognition", https://arxiv.org/abs/1512.03385

layer name	output size	18-layer	34-layer	50-layer	101-layer	152-layer
conv1	112×112	7×7, 64, stride 2				
conv2_x	56×56	3×3 max pool, stride 2				
		$\begin{bmatrix}3\times3, 64\\3\times3, 64\end{bmatrix}\times2$	$\begin{bmatrix}3\times3, 64\\3\times3, 64\end{bmatrix}\times3$	$\begin{bmatrix}1\times1, 64\\3\times3, 64\\1\times1, 256\end{bmatrix}\times3$	$\begin{bmatrix}1\times1, 64\\3\times3, 64\\1\times1, 256\end{bmatrix}\times3$	$\begin{bmatrix}1\times1, 64\\3\times3, 64\\1\times1, 256\end{bmatrix}\times3$
conv3_x	28×28	$\begin{bmatrix}3\times3, 128\\3\times3, 128\end{bmatrix}\times2$	$\begin{bmatrix}3\times3, 128\\3\times3, 128\end{bmatrix}\times4$	$\begin{bmatrix}1\times1, 128\\3\times3, 128\\1\times1, 512\end{bmatrix}\times4$	$\begin{bmatrix}1\times1, 128\\3\times3, 128\\1\times1, 512\end{bmatrix}\times4$	$\begin{bmatrix}1\times1, 128\\3\times3, 128\\1\times1, 512\end{bmatrix}\times8$
conv4_x	14×14	$\begin{bmatrix}3\times3, 256\\3\times3, 256\end{bmatrix}\times2$	$\begin{bmatrix}3\times3, 256\\3\times3, 256\end{bmatrix}\times6$	$\begin{bmatrix}1\times1, 256\\3\times3, 256\\1\times1, 1024\end{bmatrix}\times6$	$\begin{bmatrix}1\times1, 256\\3\times3, 256\\1\times1, 1024\end{bmatrix}\times23$	$\begin{bmatrix}1\times1, 256\\3\times3, 256\\1\times1, 1024\end{bmatrix}\times36$
conv5_x	7×7	$\begin{bmatrix}3\times3, 512\\3\times3, 512\end{bmatrix}\times2$	$\begin{bmatrix}3\times3, 512\\3\times3, 512\end{bmatrix}\times3$	$\begin{bmatrix}1\times1, 512\\3\times3, 512\\1\times1, 2048\end{bmatrix}\times3$	$\begin{bmatrix}1\times1, 512\\3\times3, 512\\1\times1, 2048\end{bmatrix}\times3$	$\begin{bmatrix}1\times1, 512\\3\times3, 512\\1\times1, 2048\end{bmatrix}\times3$
	1×1	average pool, 1000-d fc, softmax				
FLOPs		1.8×10^9	3.6×10^9	3.8×10^9	7.6×10^9	11.3×10^9

Table 1 of "Deep Residual Learning for Image Recognition", https://arxiv.org/abs/1512.03385

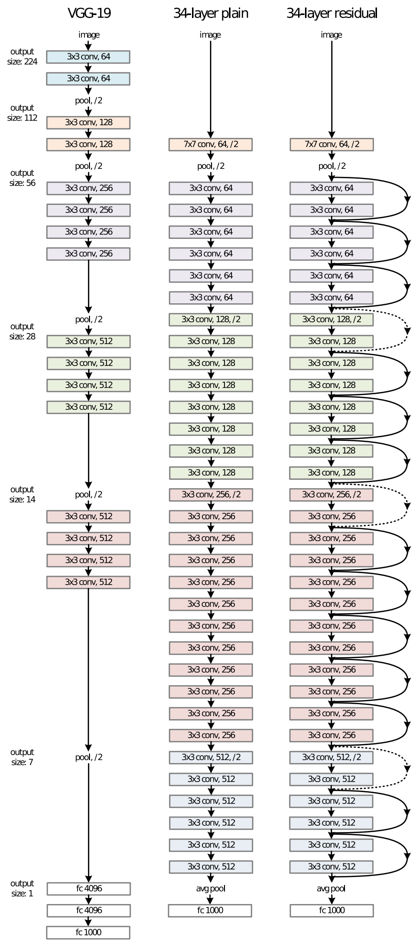


Figure 3 of "Deep Residual Learning for Image Recognition", https://arxiv.org/abs/1512.03385

The residual connections cannot be applied directly when number of channels increases.

The authors considered several alternatives, and chose the one where in case of channels increase a 1×1 convolution + BN is used on the projections to match the required number of channels. The required spacial resolution is achieved by using stride 2.

ResNet – 2015 (3.6% ILSVRC top-5 error)

dělení gradientu

hloubka už pomáhá!

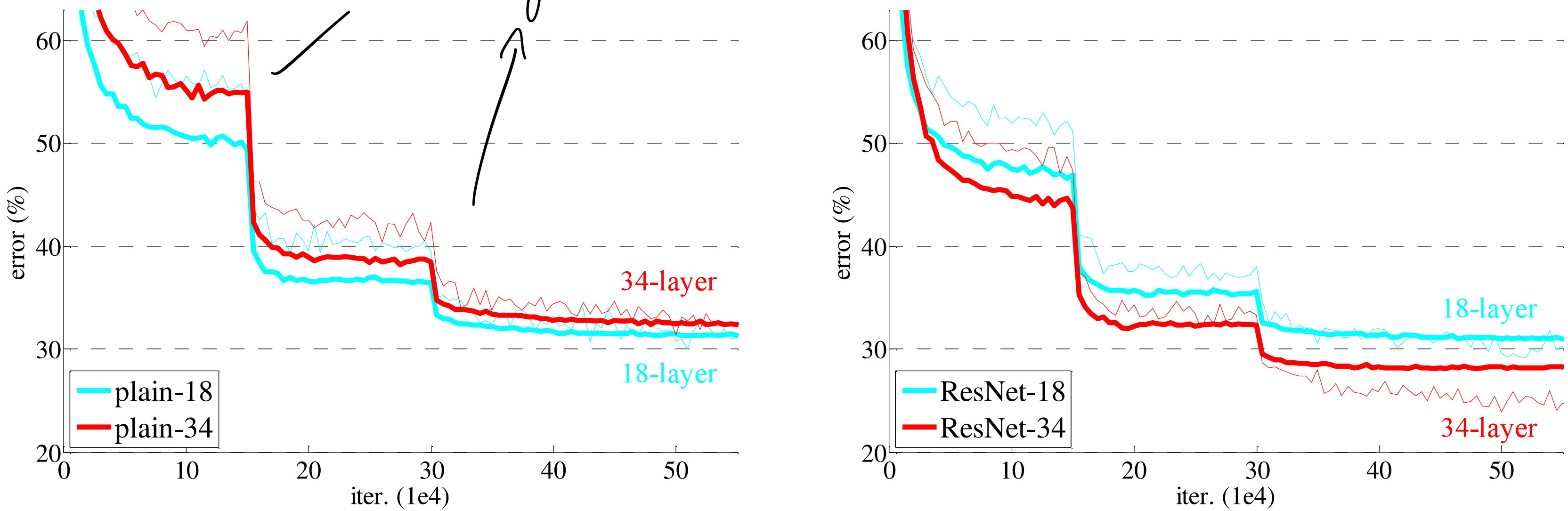


Figure 4. Training on **ImageNet**. Thin curves denote training error, and bold curves denote validation error of the center crops. Left: plain networks of 18 and 34 layers. Right: ResNets of 18 and 34 layers. In this plot, the residual networks have no extra parameter compared to their plain counterparts.

Figure 4 of "Deep Residual Learning for Image Recognition", https://arxiv.org/abs/1512.03385

ResNet – 2015 (3.6% ILSVRC top-5 error)

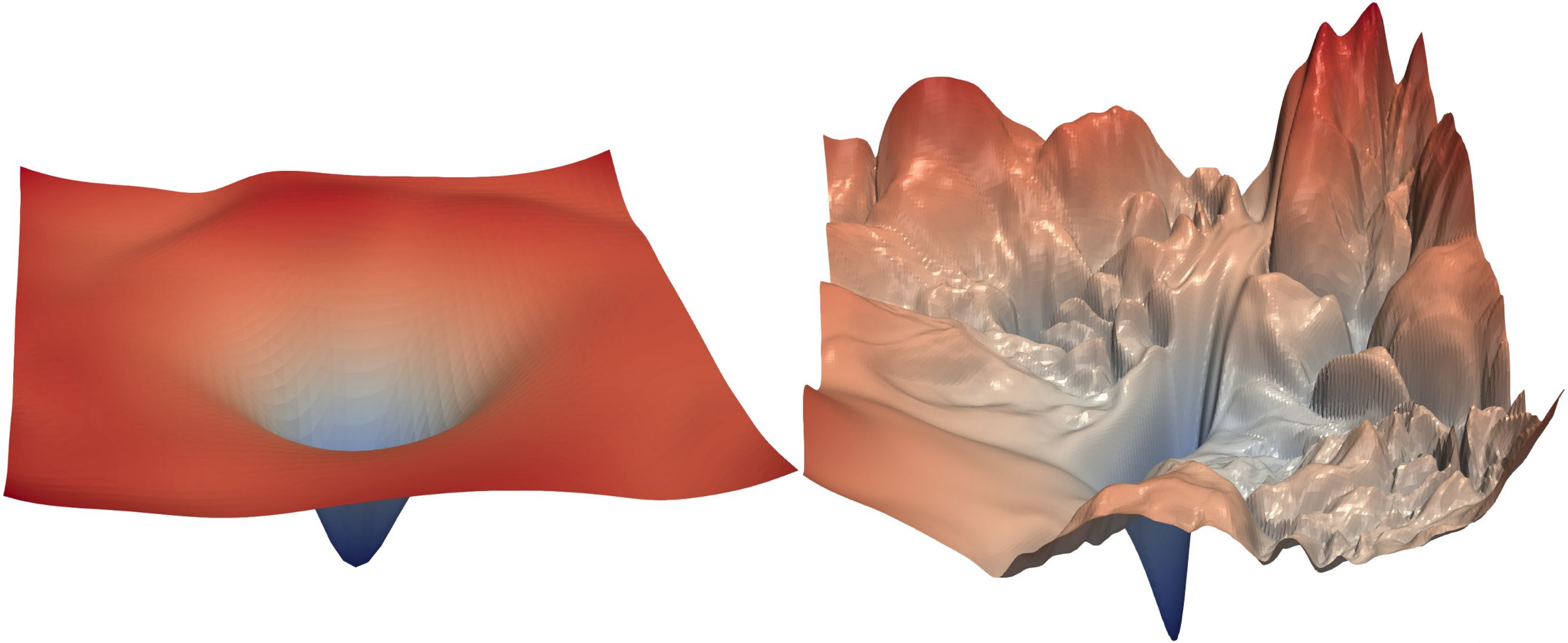


Figure 1 of "Visualizing the Loss Landscape of Neural Nets", https://arxiv.org/abs/1712.09913

ResNet – 2015 (3.6% ILSVRC top-5 error)

Training details:

- batch normalizations after each convolution and before activation
- SGD with batch size 256 and momentum of 0.9
- learning rate starts with 0.1 and "is divided by 10 when the error plateaus"
  - 600k training iterations are used (120 epochs, each containing 1.281M images)
  - according to one graph (and to their later paper) they decay at 25% and 50% of the training, so after epochs 30 and 60
    - other concurrent papers also use exponential decay or 25%-50%-75%
- no dropout, weight decay 0.0001
- during training, an image is resized with its shorter side randomly sampled in the range $[256, 480]$, and a random 224×224 crop is used
- during testing, 10-crop evaluation strategy is used
  - for the best results, the scores across multiple scales are averaged – the images are resized so that their smaller size is in $\{224, 256, 384, 480, 640\}$

method	top-1 err.	top-5 err.
VGG [41] (ILSVRC'14)	-	8.43†
GoogLeNet [44] (ILSVRC'14)	-	7.89
VGG [41] (v5)	24.4	7.1
PReLU-net [13]	21.59	5.71
BN-inception [16]	21.99	5.81
ResNet-34 B	21.84	5.71
ResNet-34 C	21.53	5.60
ResNet-50	20.74	5.25
ResNet-101	19.87	4.60
ResNet-152	**19.38**	**4.49**

Table 4. Error rates (%) of **single-model** results on the ImageNet validation set (except † reported on the test set).

Table 4 of "Deep Residual Learning for Image Recognition", https://arxiv.org/abs/1512.03385

method	top-5 err. (**test**)
VGG [41] (ILSVRC'14)	7.32
GoogLeNet [44] (ILSVRC'14)	6.66
VGG [41] (v5)	6.8
PReLU-net [13]	4.94
BN-inception [16]	4.82
ResNet (ILSVRC'15)	**3.57**

Table 5. Error rates (%) of **ensembles**. The top-5 error is on the test set of ImageNet and reported by the test server.

Table 5 of "Deep Residual Learning for Image Recognition", https://arxiv.org/abs/1512.03385

The ResNet-34 B uses the 1×1 convolution on residual connections with different number of input and output channels; ResNet-34 C uses this convolution on all residual connections. Variant B is used for ResNet-50/101/152.

Main Takeaways

- Convolutions can provide
  - local interactions in spacial/temporal dimensions
  - shift invariance
  - *much* less parameters than a fully connected layer
- Usually repeated 3×3 convolutions are enough, no need for larger filter sizes.
- When pooling is performed, double the number of channels (i.e., the first convolution following the pooling layer will have twice as many output channels).
- If your network is deep enough (the last hidden neurons have a large receptive fields), final fully connected layers are not needed, and global average pooling is enough.
- Batch normalization is a great regularization method for CNNs, allowing removal/decrease of dropout and L^2 regularization.
- Small weight decay (i.e., L^2 regularization) of usually 1e-4 is still useful for regularizing convolutional kernels.