Najděte explicitní vzorec aplikací vytvořujících funkcí:

Mikoláš Fromm: mecool

1-1) $a_0 = 1 \qquad a_n = 2a_{n-1} + 3$

f_0	f_1	f_2	f_3	$\dots$	$f(x)$
1	$2f_0+3$	$2f_1+3$	$2f_2+3$	$\dots$	$?$
0	$2f_0$	$2f_1$	$2f_2$	$\dots$	$2x\cdot f(x)$
3	3	3	3	$\dots$	$\frac{3}{1-x}$
-2	0	0	0	$\dots$	-2

$$f(x) = 2x\cdot f(x) + \frac{3}{1-x} - 2 \qquad \frac{3}{1-x} - \frac{2-2x}{1-x} = \frac{1+2x}{1-x}$$

$$f(x) - 2x\cdot f(x) = \frac{3}{1-x} - 2$$

hledám $\sum_{i=0}^{\infty} a_i x^i$

$$f(x)\cdot(1-2x) = \frac{1+2x}{1-x}$$

$$f(x) = \frac{1+2x}{1-x}\cdot\frac{1}{1-2x} = \frac{2x+1}{2x^2-3x+1} = \frac{3}{x-1} - \frac{4}{2x-1} = -\frac{3}{1-x} + \frac{4}{1-2x}$$

$$-3\cdot\frac{1}{1-x} \quad +4\cdot\frac{1}{1-2x}$$

$$= -3\left(\sum_{n=0}^{\infty} x^n\right) + 4\cdot\left(\sum_{n=0}^{\infty} 2^n x^n\right)$$

$$= -3\cdot\left(\sum_{n=0}^{\infty} x^n\right) + \left(\sum_{n=0}^{\infty} 2^{n+2} x^n\right)$$

$$= \sum_{n=0}^{\infty} \left(2^{n+2} - 3\right) x^n$$

Kontrola:

$1x^0 = a_0 = 1$

$5x^1 = a_1 = 5$

$13x^2 = a_2 = 13$

$29x^3 = a_3 = 29$ ☒

1-2) $a_0 = 1,\ a_1 = 2,\ a_n = 2a_{n-1} - a_{n-2}$

f_0	f_1	f_2	f_3	f_4	$\dots$	$f(x)$
1	2	$2f_1 - f_0$	$2f_2 - f_1$		$\dots$	$?$
0	$2f_0$	$2f_1$	$2f_2$		$\dots$	$2x\cdot f(x)$
0	0	$-f_0$	$-f_1$		$\dots$	$-x^2\cdot f(x)$
1	0	0	0		$\dots$	1

$$f(x) = 2x\cdot f(x) - x^2\cdot f(x) + 1$$

$$f(x) - 2x f(x) + x^2\cdot f(x) = 1$$

$$f(x)\cdot(x^2 - 2x + 1) = 1$$

$$f(x) = \frac{1}{x^2-2x+1} = \frac{1}{(x-1)^2} = \frac{1}{(1-x)^2} \Rightarrow \sum_{n=0}^{\infty}\binom{k-1+n}{k-1}x^n = \sum_{n=0}^{\infty}\binom{1+n}{1}x^n = \sum_{n=1}^{\infty} n x^{n-1}$$

Kontrola:

$a_0 = 1 = 1x^0$

$a_1 = 2 = 2x^1$

$a_3 = 3 = 3x^2$

$a_4 = 4 = 4x^3$

$a_5 = 5 = 5x^4$ ☒

1-3)

$a_0 = 2, \quad a_1 = 3, \quad a_n = 3a_{n-1} - 2a_{n-2}$

	f_0	f_1	f_2	f_3	f_4	$\dots$	$f(x)$
	2	3	$3f_1 - 2f_0$	$3f_2 - 2f_1$		$\dots$	
=	$\emptyset$	$3f_0$	$3f_1$	$3f_2$	$3f_3$		$3x \cdot f(x)$
+	$\emptyset$	$\emptyset$	$-2f_0$	$-2f_1$	$-2f_2$		$-2x^2 \cdot f(x)$
+	2	-3	$\emptyset$	$\emptyset$	$\emptyset$		$2-3x$

$$f(x) = 3x \cdot f(x) - 2x^2 \cdot f(x) + 2 - 3x$$

$$f(x) \cdot \left(2x^2 - 3x + 1\right) = 2 - 3x$$

$$f(x) = \frac{2-3x}{2x^2-3x+1} = \frac{2-3x}{(2x-1)\cdot(x-1)} = \frac{\alpha}{(2x-1)} + \frac{\beta}{(x-1)} = \frac{-1}{2x-1} + \frac{-1}{x-1}$$

$$f(x) = \frac{1}{1-2x} + \frac{1}{1-x} = \sum_{n=0}^{\infty} k^n x^n + \sum_{n=0}^{\infty} x^n = \sum_{n=0}^{\infty} (k^n+1)\, x^n, \quad k = 2$$

Kontrola:

$2 = (2^0+1)x^0$ $\qquad a_0 = 2$

$3 = (2^1+1)\, x^1$ $\qquad a_1 = 3$

$5 = (2^2+1)\, x^2$ $\qquad a_2 = 5$

$9 = (2^3+1)\, x^3$ $\qquad a_3 = 9$

$17 = (2^4+1)\, x^4$ $\qquad a_4 = 17$

$\boxtimes$