3D space:

Homogenous:

$$p = (x, y, z, w)^T$$

$$R = \begin{pmatrix} - & - & - & 0 \\ - & - & - & 0 \\ - & - & - & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 0 & 0 & - \\ 0 & 1 & 0 & - \\ 0 & 0 & 1 & - \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Jak by vypadala matice rotace okolo osy v 3D?

$$R_x = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) & 0 \\ 0 & \sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

→ rotuju okolo osy x, tudíž x-ový parametr se mi nemůže změnit. Tudíž $(1,0,0,0)^T$

$$R_y = \begin{pmatrix} \cos(\alpha) & 0 & \sin(\alpha) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\alpha) & 0 & \cos(\alpha) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

→ y musí zůstat netknutý

→ znaménka u sinus jsou ale obráceně, protože jdu od osy z k ose x.

$$R_z = \begin{pmatrix} \cos(\alpha) & -\sin(\alpha) & 0 & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Všechno najednou?

osu x o α, osu y o β, osu z o γ

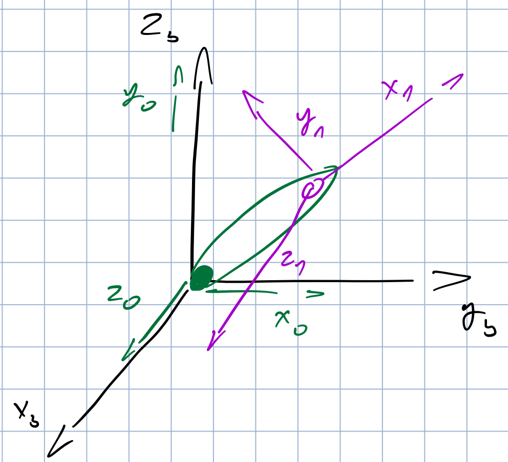
$$R = \begin{pmatrix} c(\beta)c(\gamma) & -c(\beta)c(\gamma) & \sin(\beta) \\ s(\alpha)s(\beta)c(\gamma) + c(\alpha)s(\gamma) & -s(\alpha)s(\beta)s(\gamma) + c(\alpha)c(\gamma) & -s(\alpha)c(\beta) \\ -c(\alpha)s(\beta)c(\gamma) + s(\alpha)s(\gamma) & c(\alpha)s(\beta)s(\gamma) + s(\alpha)c(\gamma) & c(\alpha)c(\beta) \end{pmatrix}$$

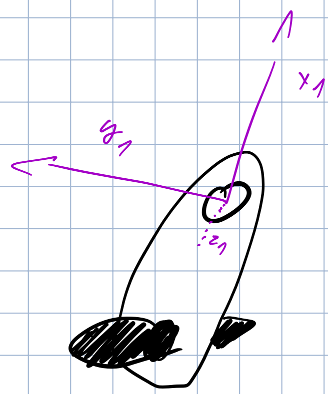
Denavit – Hartenberg:

1) Rotate along old z axis
2) Move along old z axis by the link length.
3) Displace the new link origin to its destination
4) Rotate the new z axis along new x axis

But there is need to name the axis correctly:

1) z is always the axis of the movement
2) x_i is always $\perp$ to z_i and z_{i-1}
3) y is always set by the right hand rule.
4) x_i is intersecting z_{i-1}



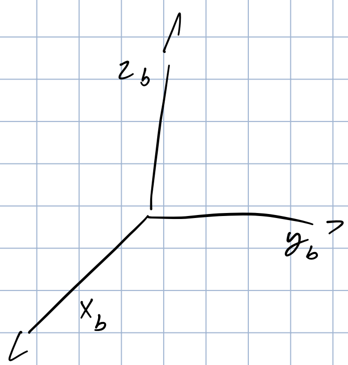


parametr může být pouze ve dvou sloupcích

	θ	d	a	α
0	$\pi/2$	0	0	$\pi/2$
1	q_1	0	l_1	0

transformuje „base" souřadný systém na to, co chceme.

<=> Tabulka pro ten jednoduchý joint



	θ	d	a	α
0	0	l_0	0	0
1	q_1	0	0	0
2	$\pi/2$	q_2	0	$\pi/2$
3	0	q_3	0	0

DU: Vykreslete souřadnice v čase $t = 0 \dots 3$ pro jemný Δt pro koncovku tohoto manipulátoru.

Matice přechodu A_{i-1}^{i} pro LCS_i:

- $A_{i-1}^{i} = A_{z_{i-1},\vartheta_i} \cdot A_{z_{i-1},d_i} \cdot A_{x,a_i} \cdot A_{x,\alpha_i}$
- $A_{i-1}^{i} = \begin{bmatrix} \cos(\vartheta_i) & -\sin(\vartheta_i)\cos(\alpha_i) & \sin(\vartheta_i)\sin(\alpha_i) & a_i\cos(\vartheta_i) \\ \sin(\vartheta_i) & \cos(\vartheta_i)\cos(\alpha_i) & -\cos(\vartheta_i)\sin(\alpha_i) & a_i\sin(\vartheta_i) \\ 0 & \sin(\alpha_i) & \cos(\alpha_i) & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$
- DH parametry: $\vartheta_i, d_i, a_i, \alpha_i$
  - ϑ_i úhel mezi osami x kolem z_{i-1}
  - d_i vzdálenost mezi osami x
  - a_i vzdálenost mezi osami z
  - α_i úhel mezi osami z kolem x_i

DH tabulka, potřebuji tedy složit 4 tyto matice ↗

	ϑ	d	a	α
0	0	l_0	0	0
1	q_1	0	0	0
2	$\pi/2$	q_2	0	$\pi/2$
3	0	q_3	0	0

$$A_0^1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad A_1^2 = \begin{pmatrix} \cos(q_1) & -\sin(q_1) & 0 & 0 \\ \sin(q_1) & \cos(q_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A_2^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & q_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad A_3^4 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Jeden krok je tedy: $A_0^1 \cdot A_1^2 \cdot A_2^3 \cdot A_3^4 =$

$$\begin{pmatrix} -\sin(q_1) & 0 & \cos(q_1) & q_3 \cdot \cos(q_1) \\ \cos(q_1) & 0 & \sin(q_1) & q_3 \cdot \sin(q_1) \\ 0 & 1 & 0 & q_2 + l_0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \text{Nová poloha end pointu.}$$