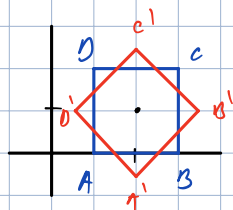


Rotation matrix:

$$\begin{pmatrix} \cos(\alpha) & -\sin(\alpha) & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Why, to go: $T \cdot R \cdot T^{-1}$ $\rightarrow$ move to (0,0,0), turn, move back

1 center = $(2, 1, 1)^T$
|edge| = 2



$$\begin{aligned} A &= (1, 0, 1)^T \\ B &= (3, 0, 1)^T \\ C &= (3, 2, 1)^T \\ D &= (1, 2, 1)^T \end{aligned}$$

$$M = \begin{pmatrix} A & B & C & D \\ 1 & 3 & 3 & 1 \\ 0 & 0 & 2 & 2 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

$$T = \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$T^{-1} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} A' & B' & C' & D' \\ 2 & \sqrt{2}+2 & 2 & -\sqrt{2}+2 \\ -\sqrt{2}+1 & 1 & \sqrt{2}+1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

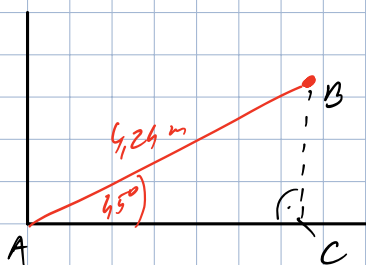
$$M \cdot (*) = (A'^T B'^T C'^T D'^T)$$

$$M = T^{-1} \cdot R \cdot T$$

$$R = \begin{pmatrix} \cos(45^\circ) & -\sin(45^\circ) & 0 \\ \sin(45^\circ) & \cos(45^\circ) & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad T \cdot R = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos \frac{\pi}{4} & -\sin \frac{\pi}{4} & 0 \\ \sin \frac{\pi}{4} & \cos \frac{\pi}{4} & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \cos \frac{\pi}{4} & -\sin \frac{\pi}{4} & 2 \\ \sin \frac{\pi}{4} & \cos \frac{\pi}{4} & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R \cdot T = \begin{pmatrix} \cos \frac{\pi}{4} & -\sin \frac{\pi}{4} & 2 \\ \sin \frac{\pi}{4} & \cos \frac{\pi}{4} & 1 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \cos \frac{\pi}{4} & -\sin \frac{\pi}{4} & -2\cos \frac{\pi}{4} + \sin \frac{\pi}{4} + 2 \\ \sin \frac{\pi}{4} & \cos \frac{\pi}{4} & -2\sin \frac{\pi}{4} - \cos \frac{\pi}{4} + 1 \\ 0 & 0 & 1 \end{pmatrix} = M$$

2



$$A = [0, 0]$$

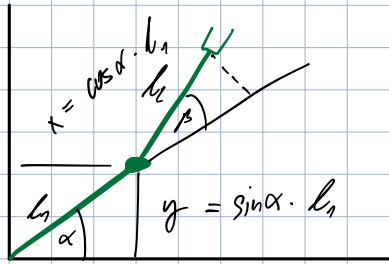
$$B = [x, y]$$

$$\sin 45^\circ = \frac{|BC|}{|AB|} \quad y = \sin 45^\circ \cdot 4.24 = 2.998$$

$$\cos 45^\circ = \frac{|AC|}{|AB|} = x \quad x = \cos 45^\circ \cdot 4.24 = 2.998$$

3

Transformation matrix for 2D RR manipulator:



$$\theta_1 = \alpha, \theta_2 = \beta \quad \rightarrow \text{pro lepší zápis}$$

$T =$ „Move Arm Joint To Origin“ • „Rotate The Joint“ • „Move It Back“ • „Rotate Around Origin“

R

R

R^{-1}

A

$$T = \begin{pmatrix} 1 & 0 & \cos \alpha \cdot l_1 \\ 0 & 1 & \sin \alpha \cdot l_1 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos(\alpha + \beta) & -\sin(\alpha + \beta) & \cos(\alpha + \beta) \cdot l_2 \\ \sin(\alpha + \beta) & \cos(\alpha + \beta) & \sin(\alpha + \beta) \cdot l_2 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & -\cos \alpha \cdot l_1 \\ 0 & 1 & -\sin \alpha \cdot l_1 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos(\alpha) & -\sin(\alpha) & \cos \alpha \cdot l_1 \\ \sin(\alpha) & \cos(\alpha) & \sin \alpha \cdot l_1 \\ 0 & 0 & 1 \end{pmatrix}$$

$\vdots$

$$T = \begin{pmatrix} \cos(\alpha) \cos(\alpha + \beta) - \sin(\alpha) \cdot \sin(\alpha + \beta) & -\sin(\alpha) \cos(\alpha + \beta) - \cos(\alpha) \cdot \sin(\alpha + \beta) & l_1 \cdot \cos(\alpha) + l_2 \cdot \cos(\alpha + \beta) \\ \cos(\alpha) \sin(\alpha + \beta) + \sin(\alpha) \cdot \cos(\alpha + \beta) & \cos(\alpha) \cos(\alpha + \beta) - \sin(\alpha) \sin(\alpha + \beta) & l_1 \cdot \sin(\alpha) + l_2 \cdot \sin(\alpha + \beta) \\ 0 & 0 & 1 \end{pmatrix}$$