MOTION MODEL

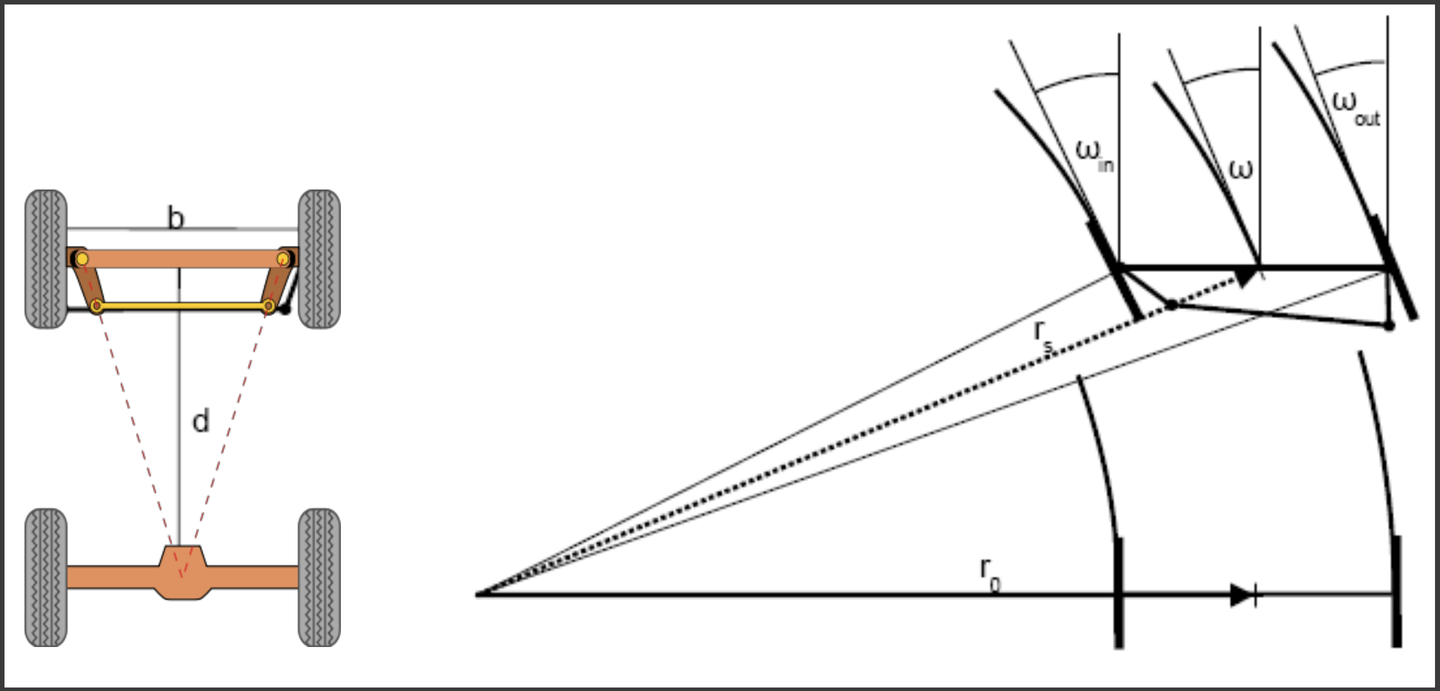
Holonomic / non-holonomic drive

Def:

A vehicle is holonomic if the number of local degrees of freedom of movement equals the number of global degrees of freedom.

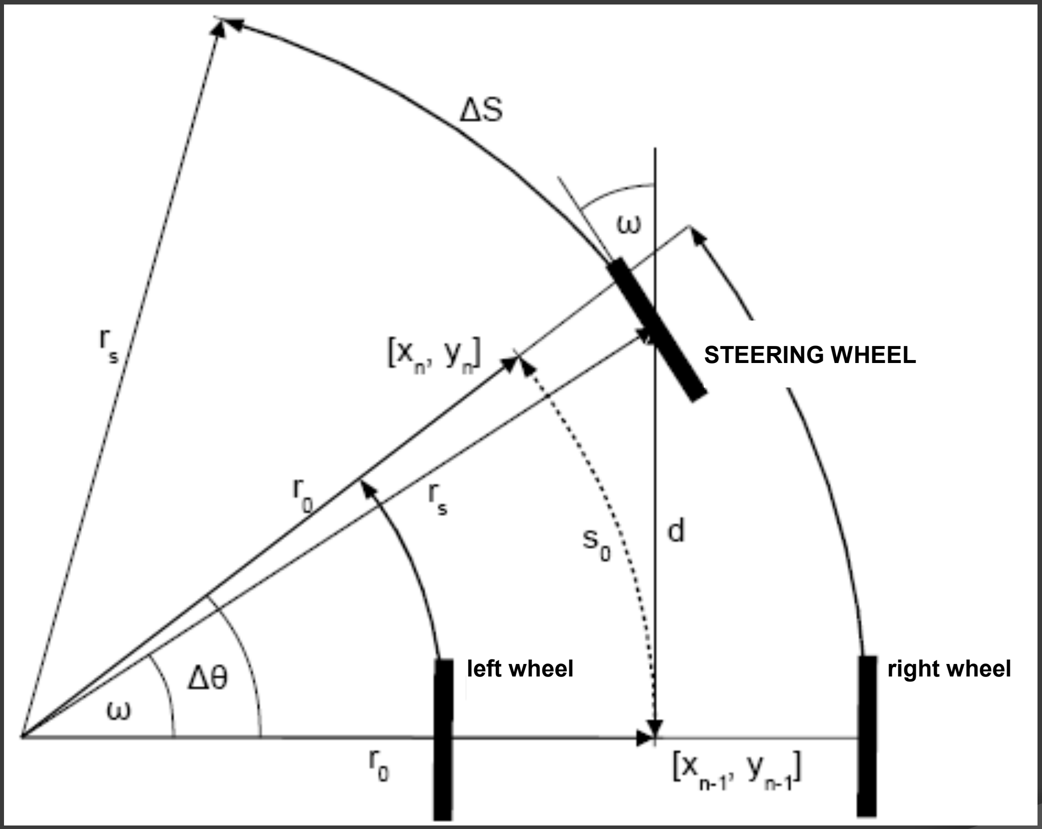
Ackermann steering

- Basic principle



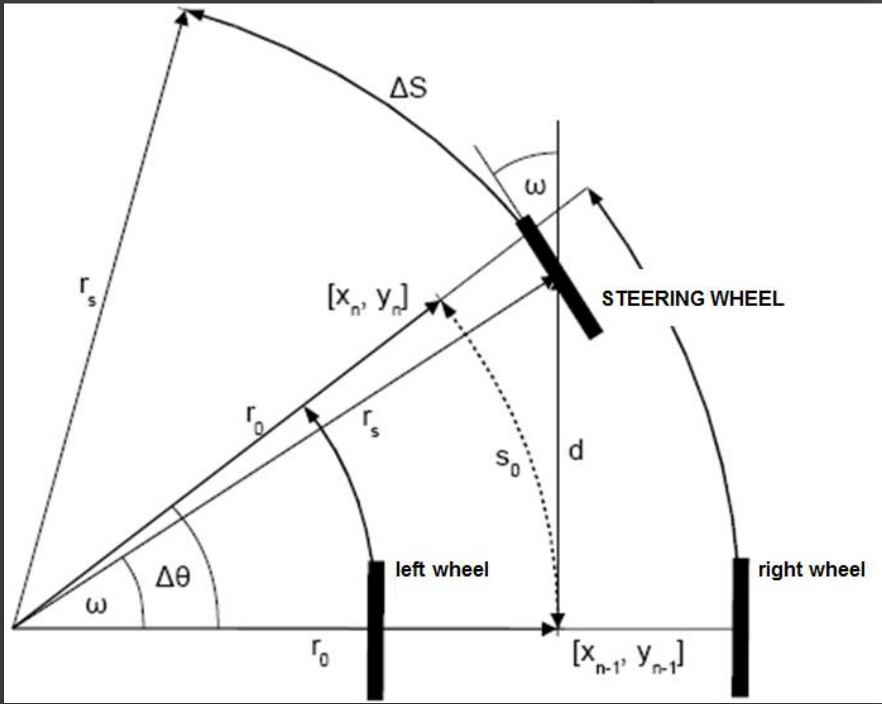
Ackermann steering

- Simplified model



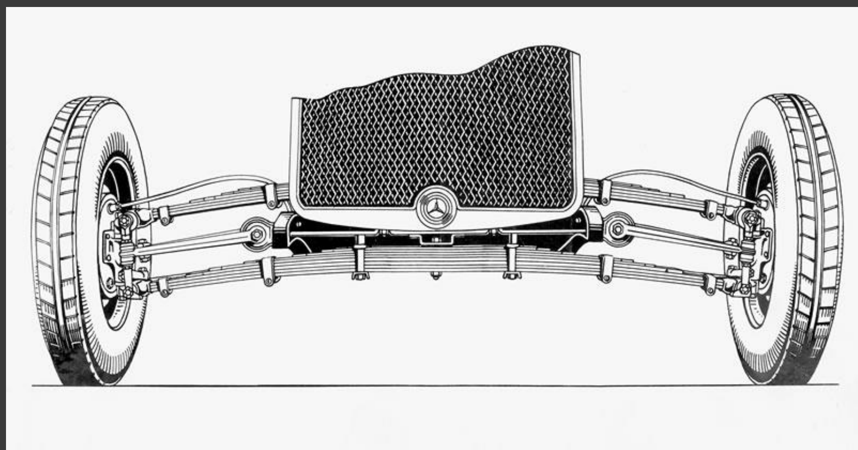
Ackermann steering

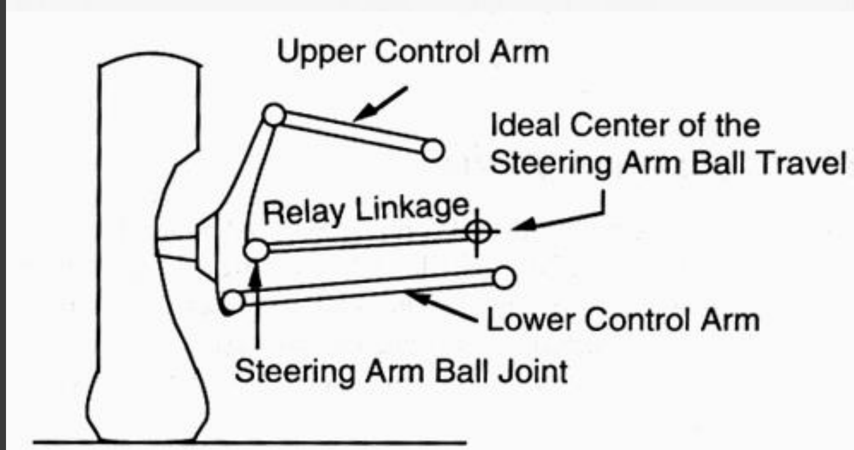
- Non-holonomic
- Constant ω ⇨ movement along a circle or straight line
- Circle: $r_s = \frac{d}{\sin \omega}$, $r_0 = \frac{d}{\operatorname{tg} \omega}$
- Orientation change $\Delta\theta = \frac{\Delta S}{r_s}$
- Manoeuvring abilities depend on d and ω_{max} (wheelbase and wheel turn)

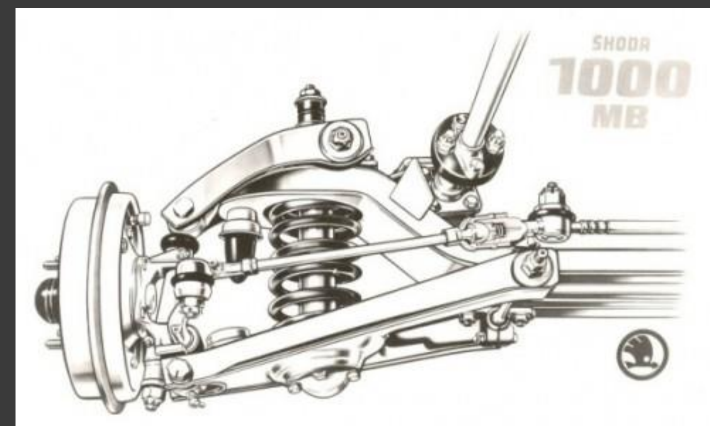


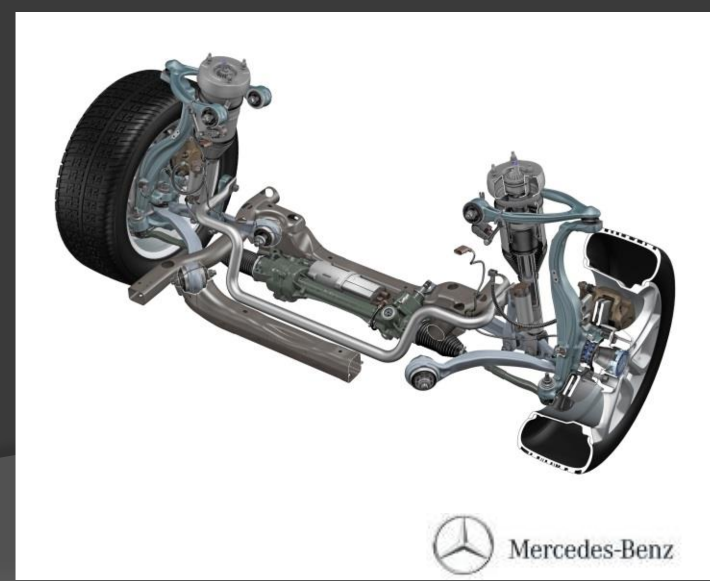
Ackermann steering

- …a little bit more complicated in real

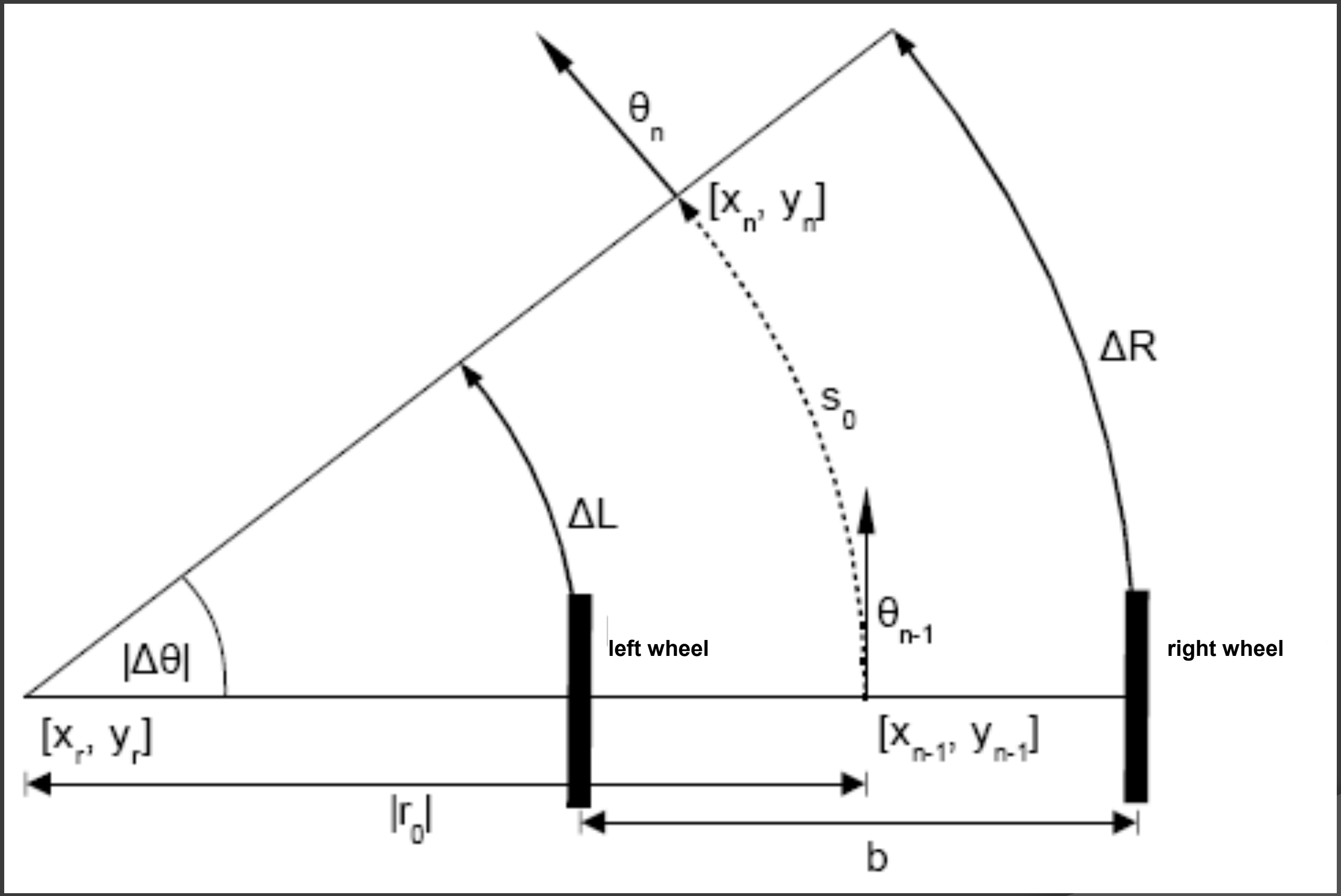






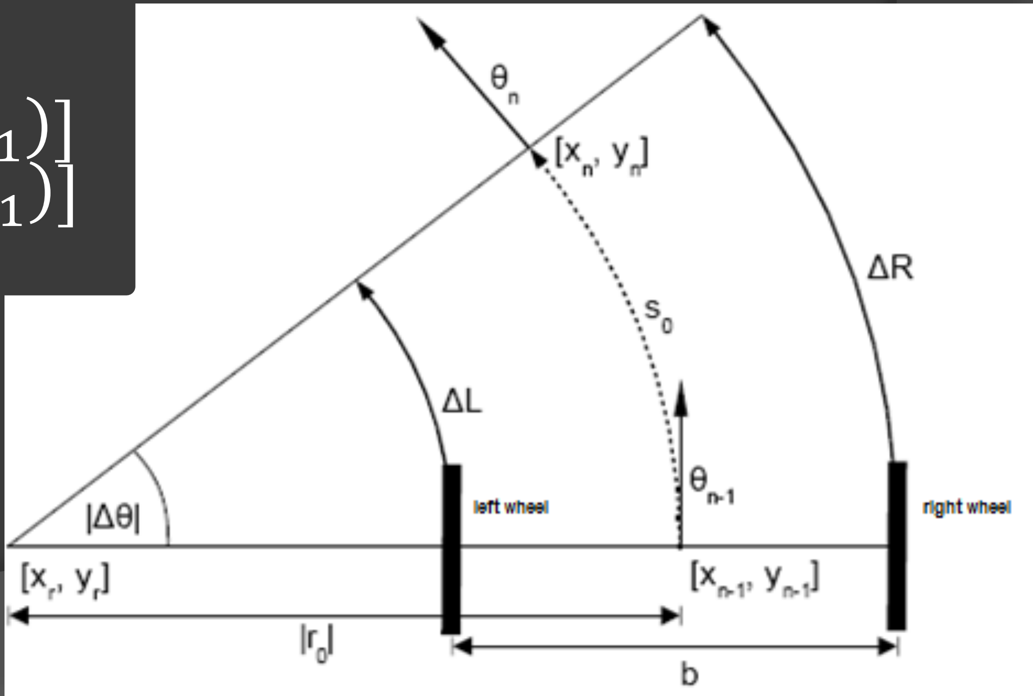


Differential steering



Differential steering

- non-holonomic
- $r_n = \frac{b}{2}\frac{\Delta R + \Delta L}{\Delta R - \Delta L}$
- Approximation:
- $\Delta\theta = \frac{s_n}{r_n} = \frac{\frac{\Delta R + \Delta L}{2}}{\frac{b}{2}\frac{\Delta R + \Delta L}{\Delta R - \Delta L}} = \frac{\Delta R - \Delta L}{b}$
- $x_n = x_{n-1} + s_n[\cos(\Delta\theta + \theta_{n-1})]$
  $y_n = y_{n-1} + s_n[\sin(\Delta\theta + \theta_{n-1})]$



Differential steering

- more precisely, integrating:

$$x(t) = x_0 + \frac{b(v_r + v_l)}{2(v_r - v_l)}\left[\sin\big((v_r - v_l)t/b + \theta\big) - \sin(\theta)\right]$$

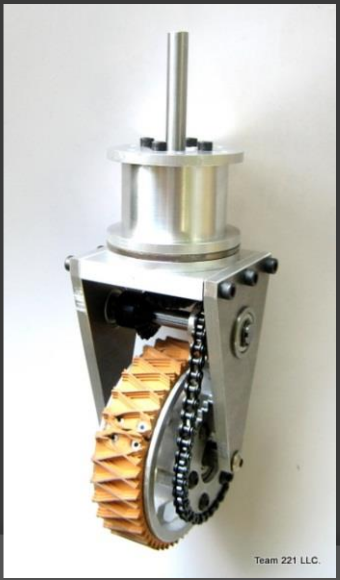
$$y(t) = y_0 + \frac{b(v_r + v_l)}{2(v_r - v_l)}\left[\cos\big((v_r - v_l)t/b + \theta\big) - \cos(\theta)\right]$$

Differential steering for real life

- General trajectory is replaced by a series of
  a) line segments ($\Delta R = \Delta L, r = \infty$)
  b) arcs $\left(\Delta R \neq \Delta L, r = \frac{b}{2}\frac{\Delta R+\Delta L}{\Delta R-\Delta L}\right)$
- Wisely set Δt
- Odometry

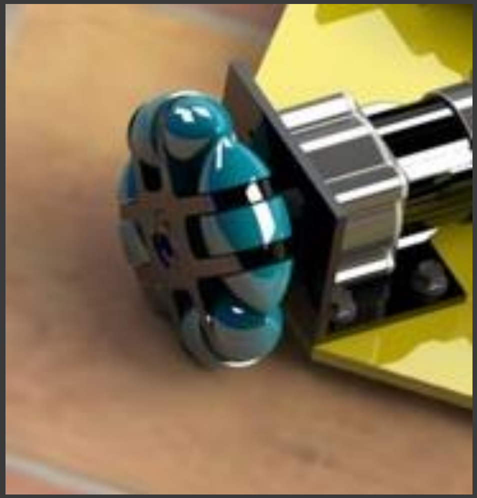
Omnidirectional steering

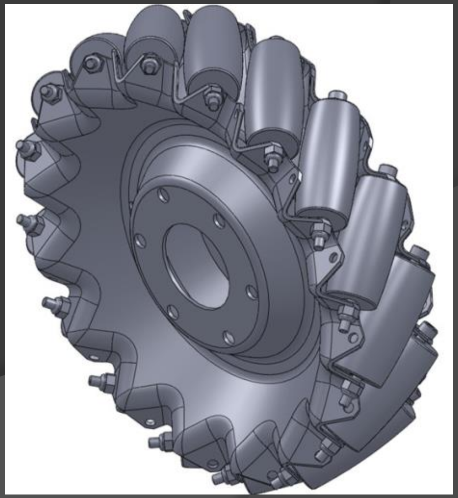
- Wheels are able to run in any direction
  - Swerve/Crab drive
  - Omniwheel
  - Mecanum wheel






Omniwheels





Syncro drive

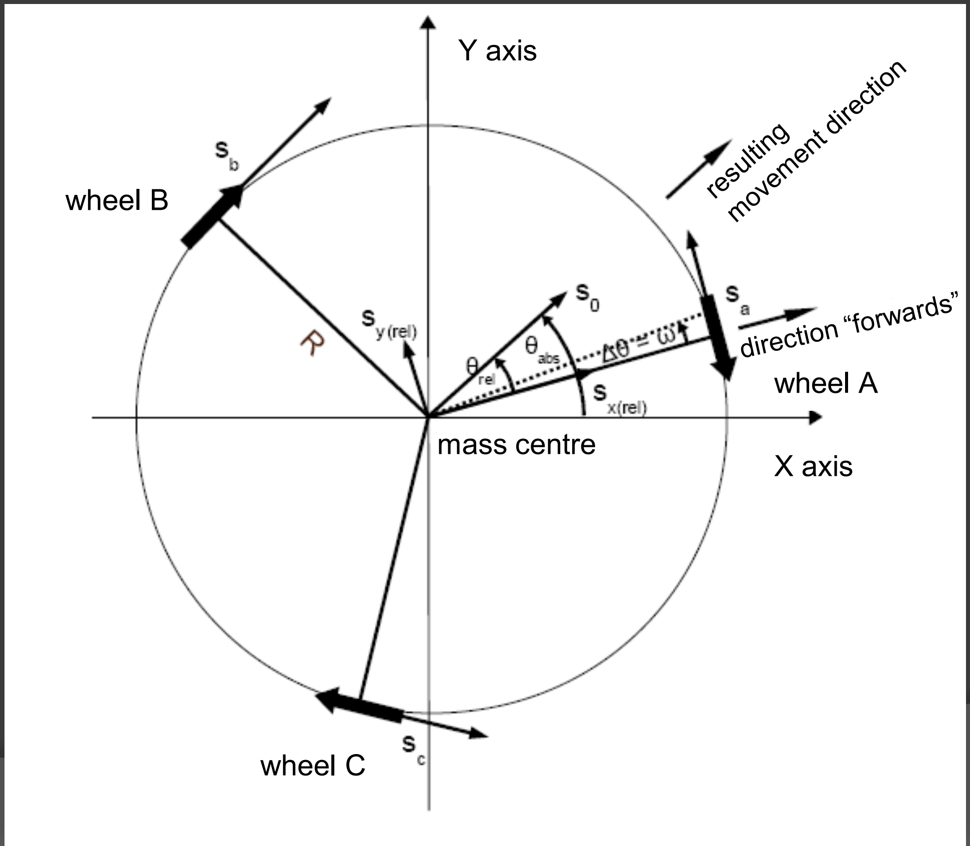
- Holds yaw (body rotation) independently to movement direction
- Arbitrary movement as for omnidrive

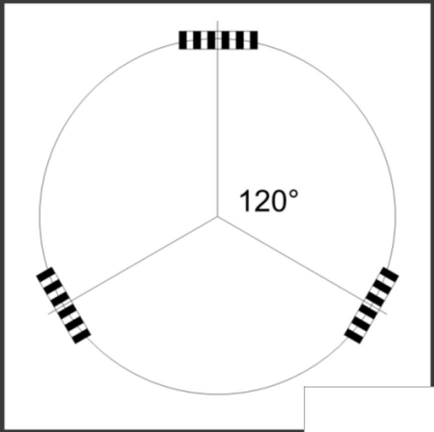
Omnidirectional steering

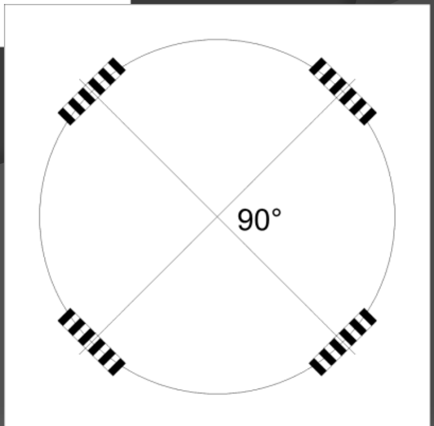
- High manoeuvring ability
  - sideways run
  - start in any direction
  - body orientation independent on movement direction
  - on-place turning
- Simpler mechanical construction for omniwheels and mecanum wheels (fixed mount)

Omnidirectional steering

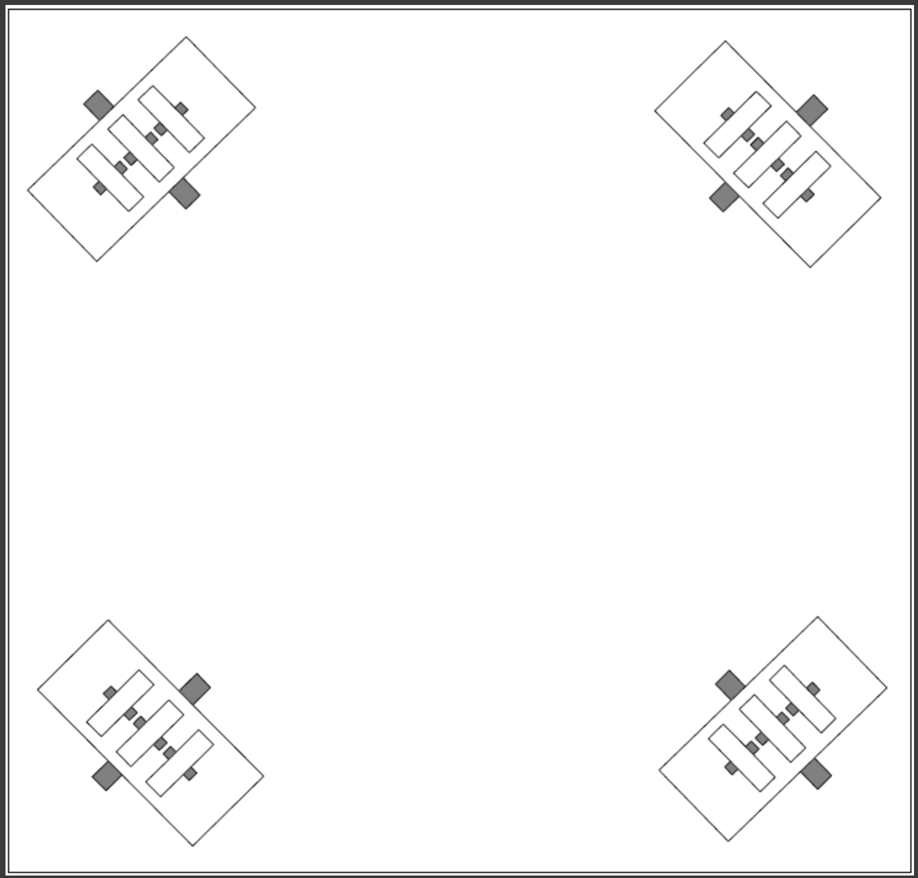
- Holonomic
- Movement easily calculated by vector combination

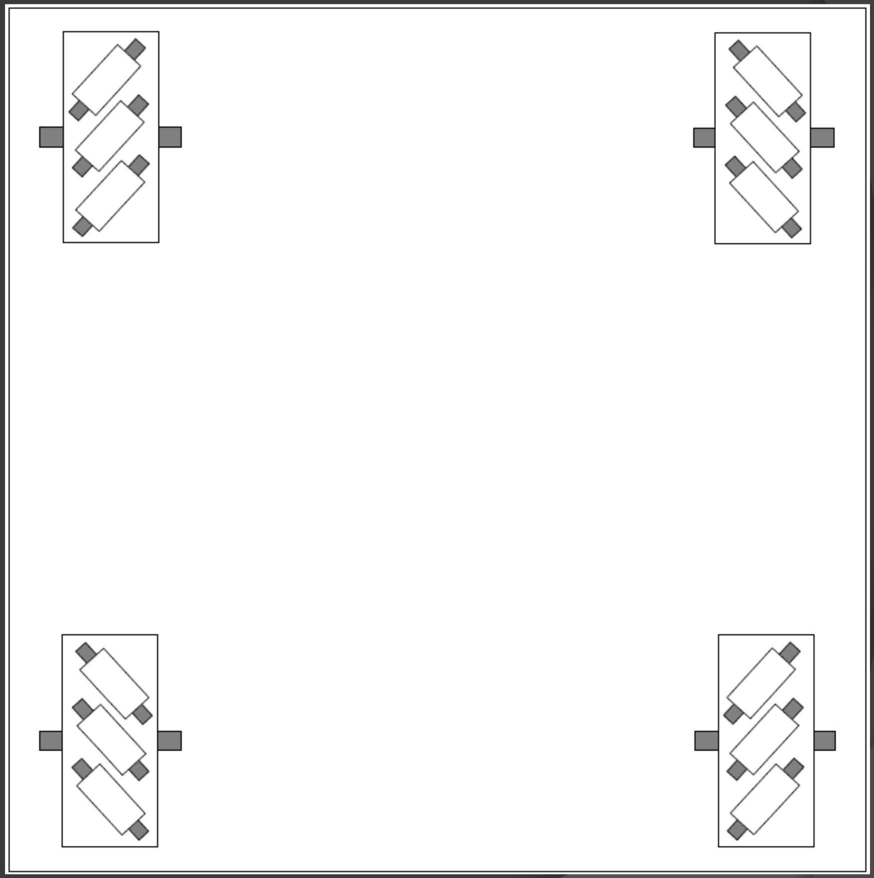




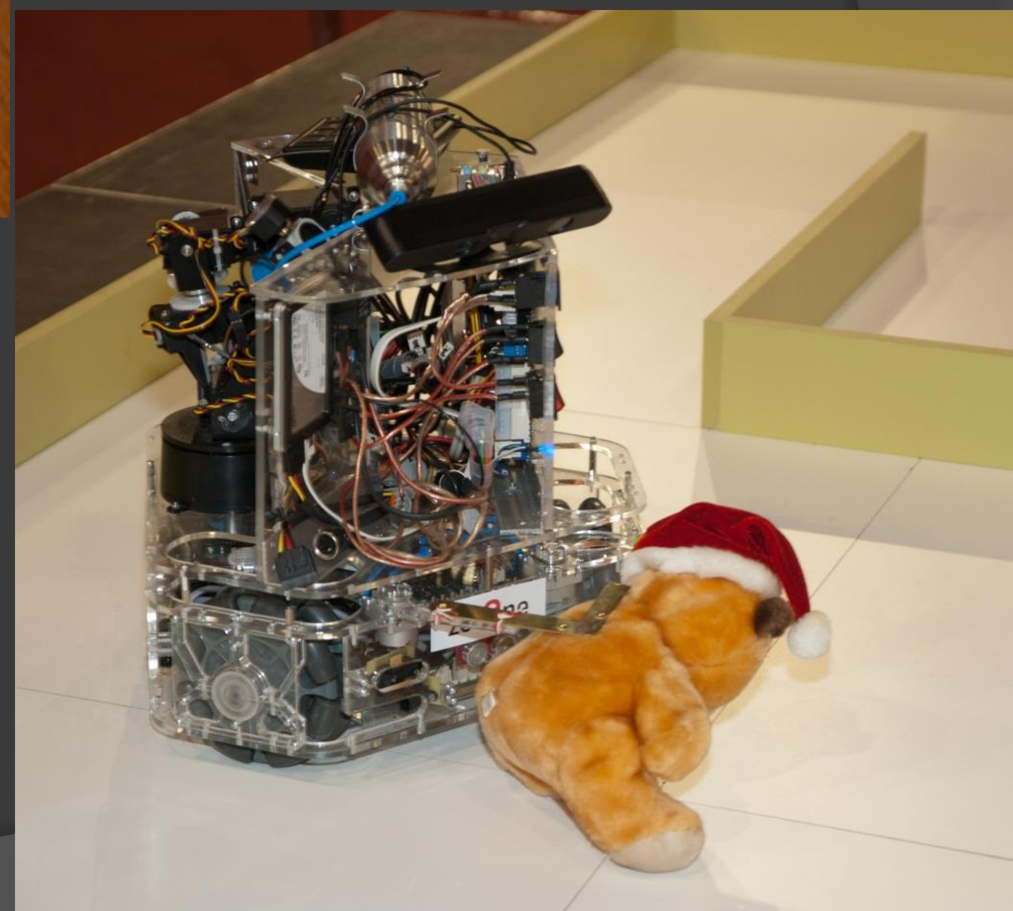


Killough / Ilon wheels

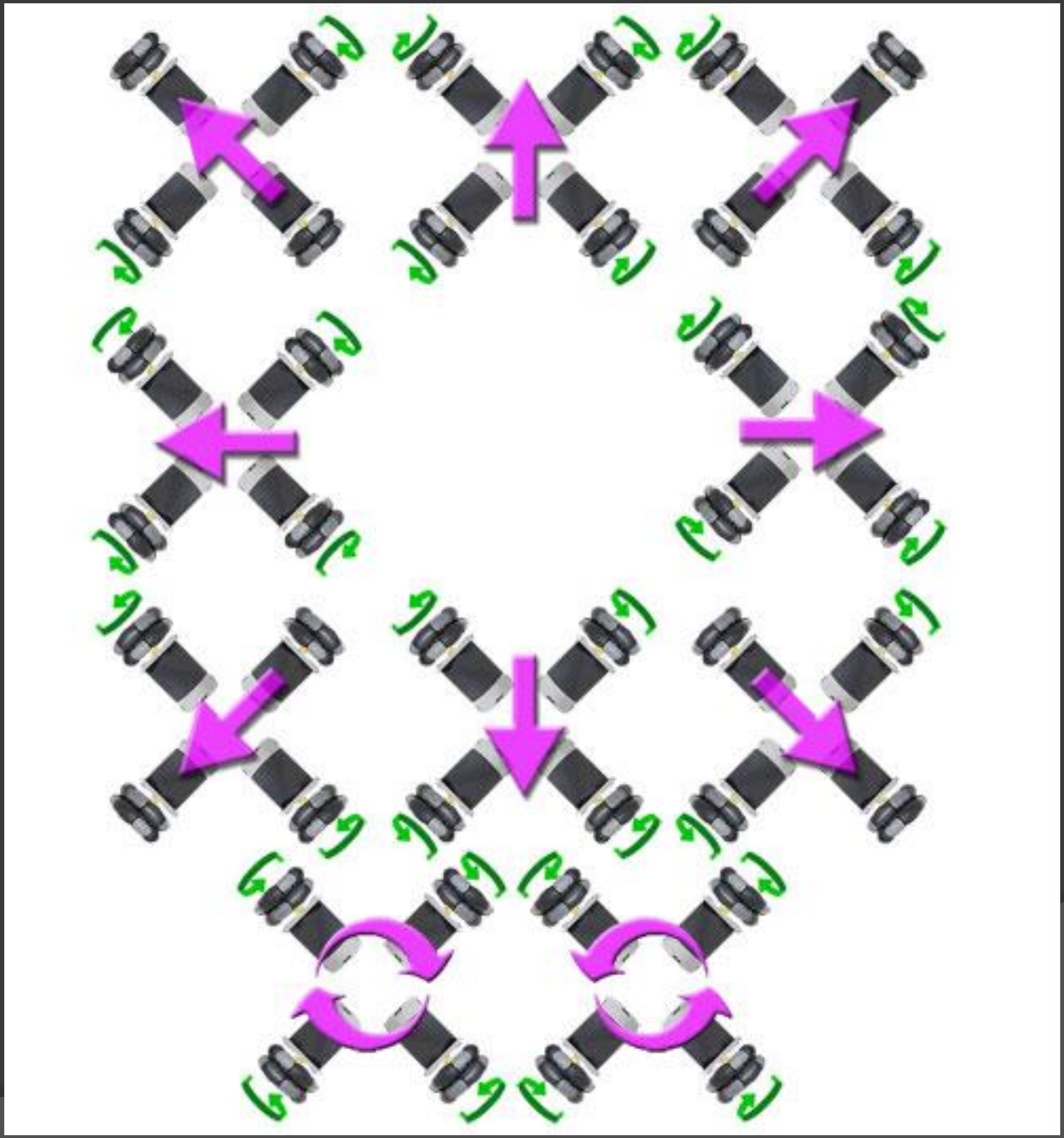




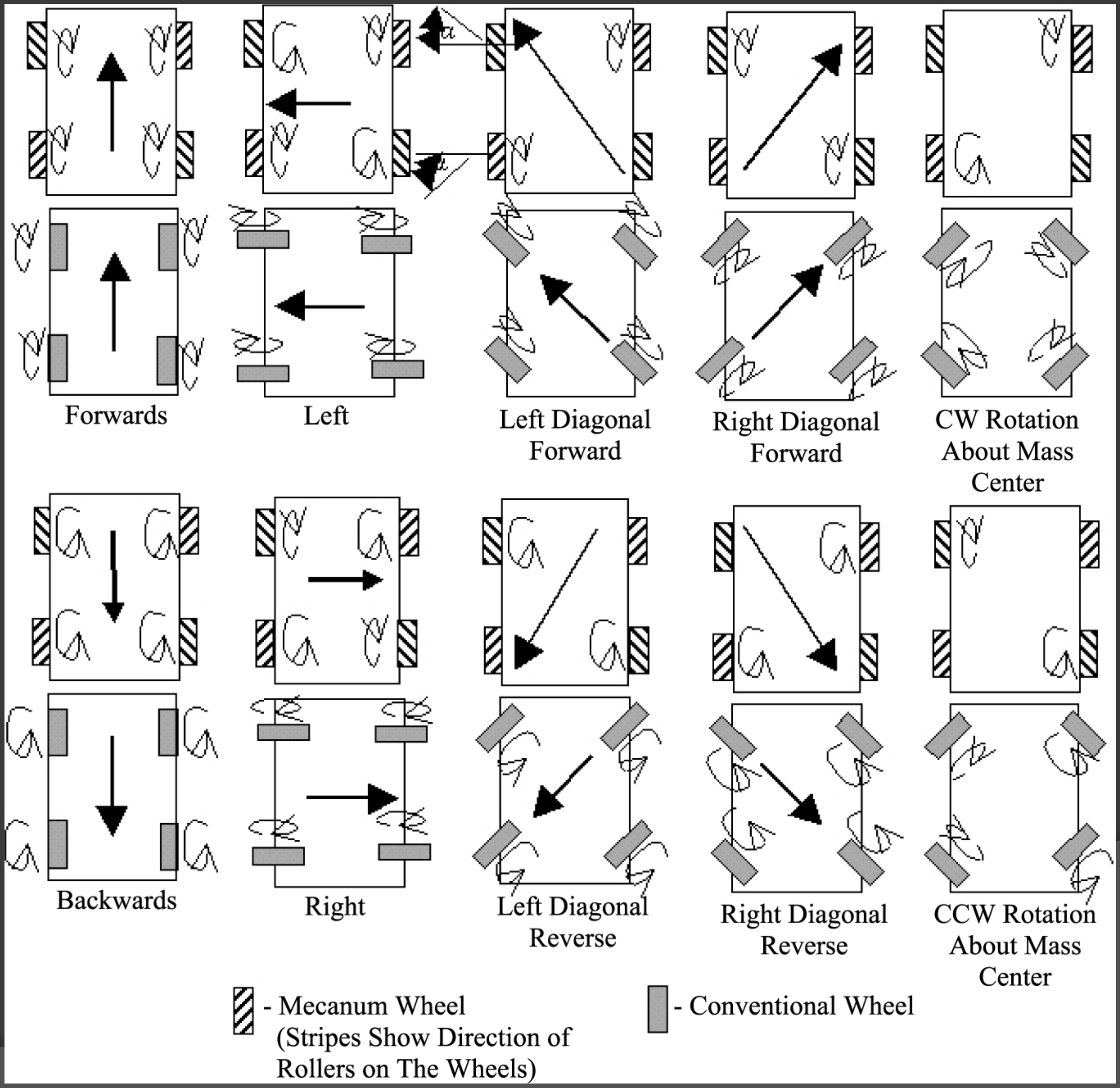

eneloop



Omniwheel drive



Mecanum & swerve drive



Robot movement

- Got $\overrightarrow{v_t}$ (translation speed) and $\vec{\omega}$ (rotation speeds)
- Need $\vec{v}$ – specific point speed

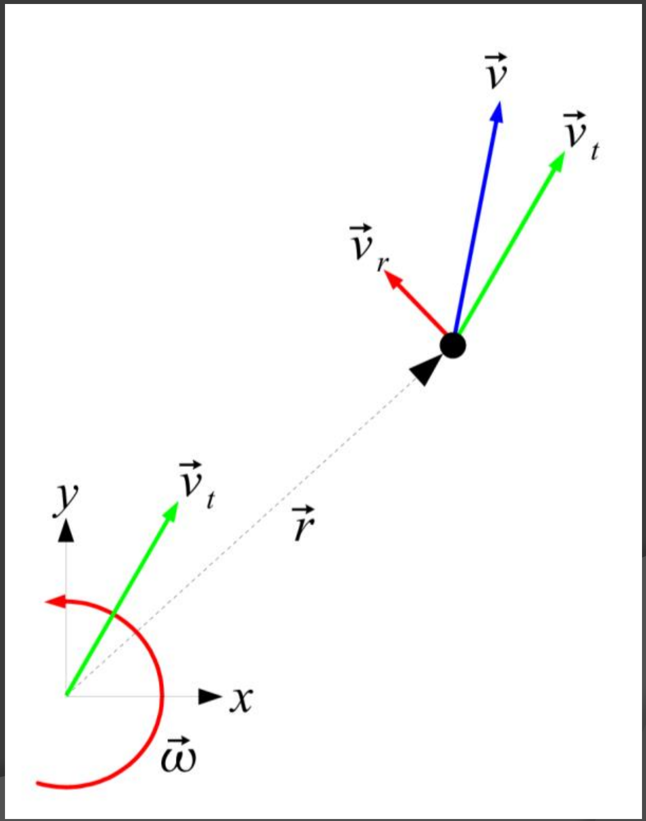
- vector approach

$$\vec{v} = \overrightarrow{v_t} + \vec{\omega} \times \vec{r}$$

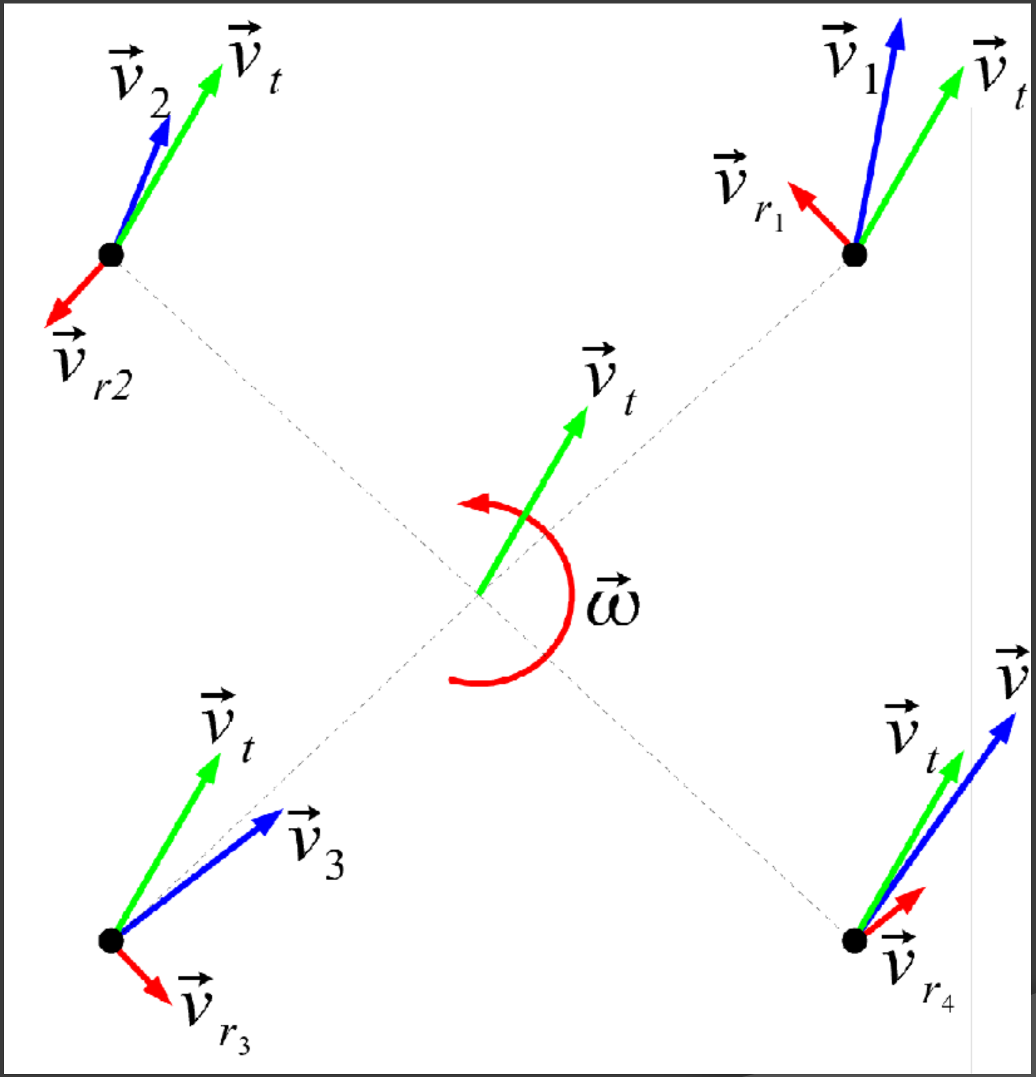
- scalar approach

$$v_x = v_{t_x} - \omega \cdot r_y$$

$$v_y = v_{t_y} + \omega \cdot r_x$$



Robot movement

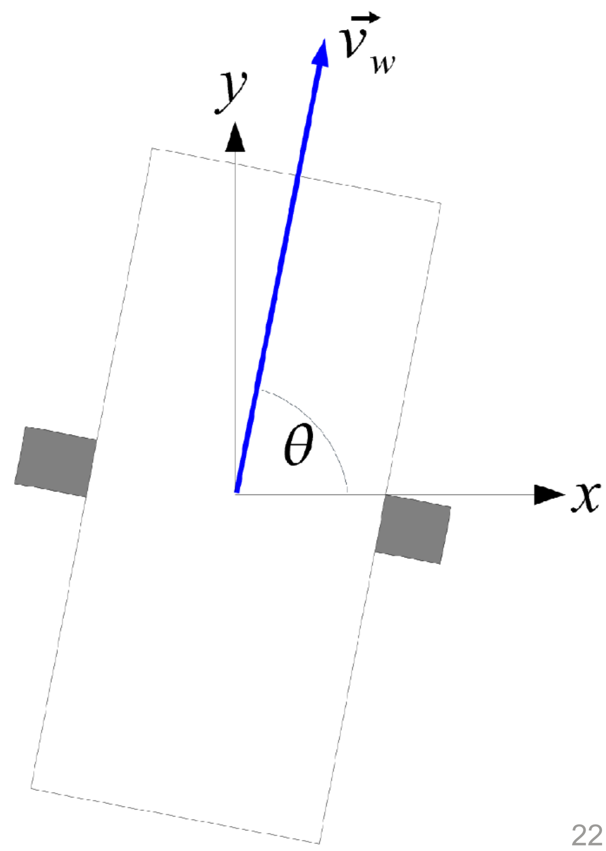


Swerve drive

- Resolve $\overrightarrow{v_t}$ (x, y components = axes velocities) into wheel speed v_w and steering angle θ

$$v_\omega = \sqrt{v_x^2 + v_y^2}$$

$$\theta = \arctan(\frac{v_y}{v_x})$$

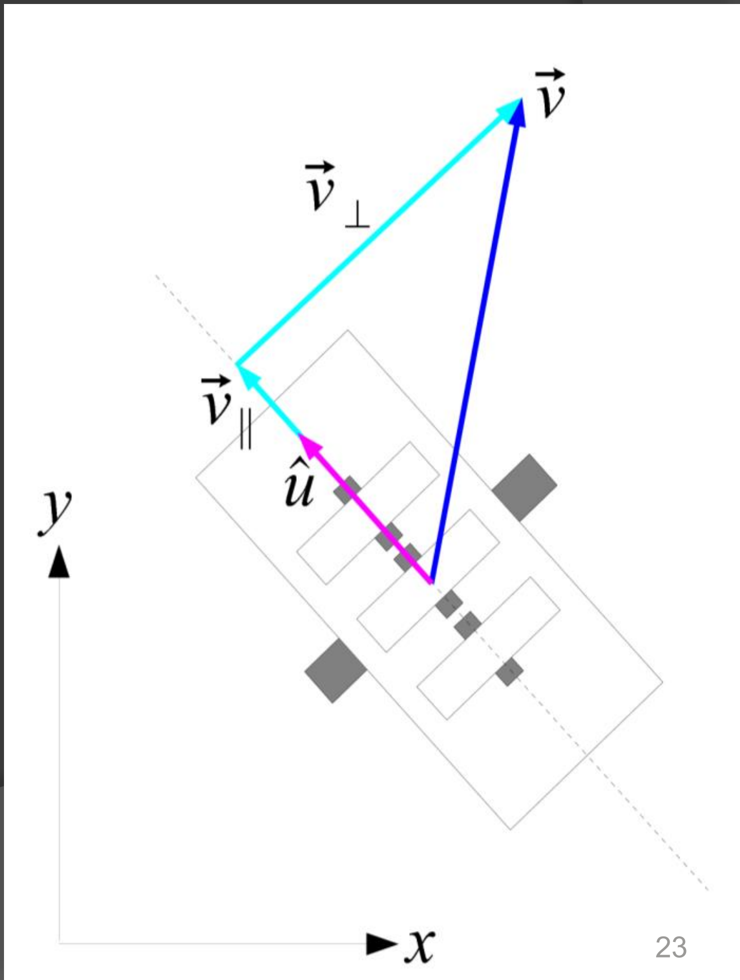


Omniwheel drive

- Resolve velocity into parallel and perpendicular components; magnitude v of parallel component is wheel speed v_w
- $\hat{u}$ is a unit vector in the direction of the wheel (whichever direction is assumed to be “forwards”)
- $v_w = v_\parallel = \vec{v} \cdot \hat{u}$

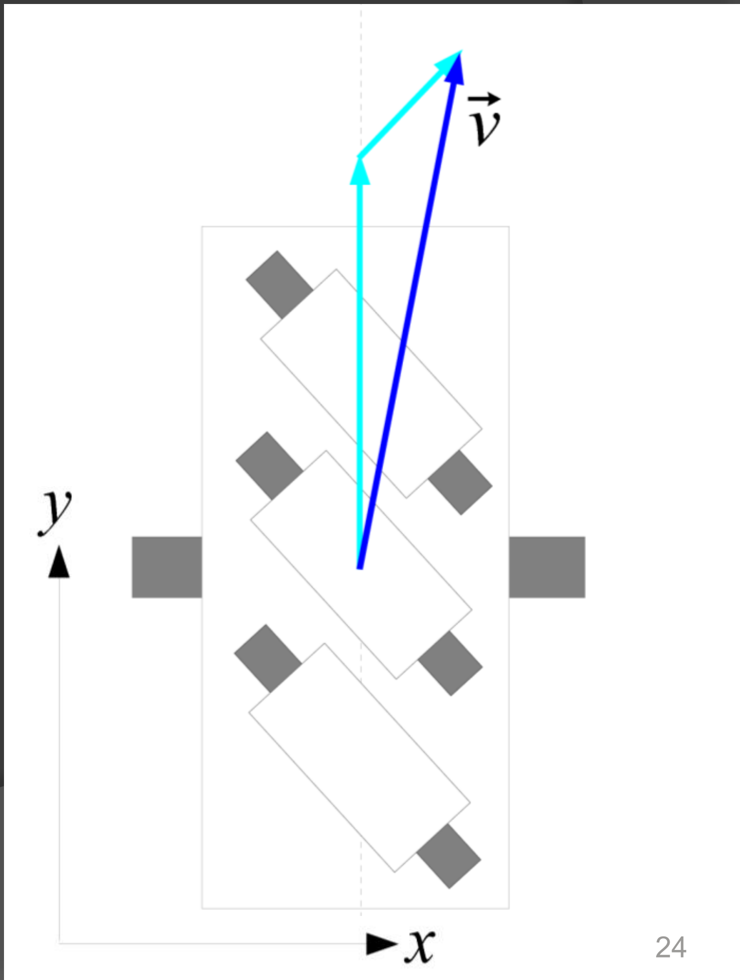
$$= (v_x \hat{\imath} + v_y \hat{\jmath}) \cdot (-\frac{1}{\sqrt{2}} \hat{\imath} + \frac{1}{\sqrt{2}} \hat{\jmath})$$

$$= -\frac{1}{\sqrt{2}} v_x + \frac{1}{\sqrt{2}} v_y$$



Mecanum drive

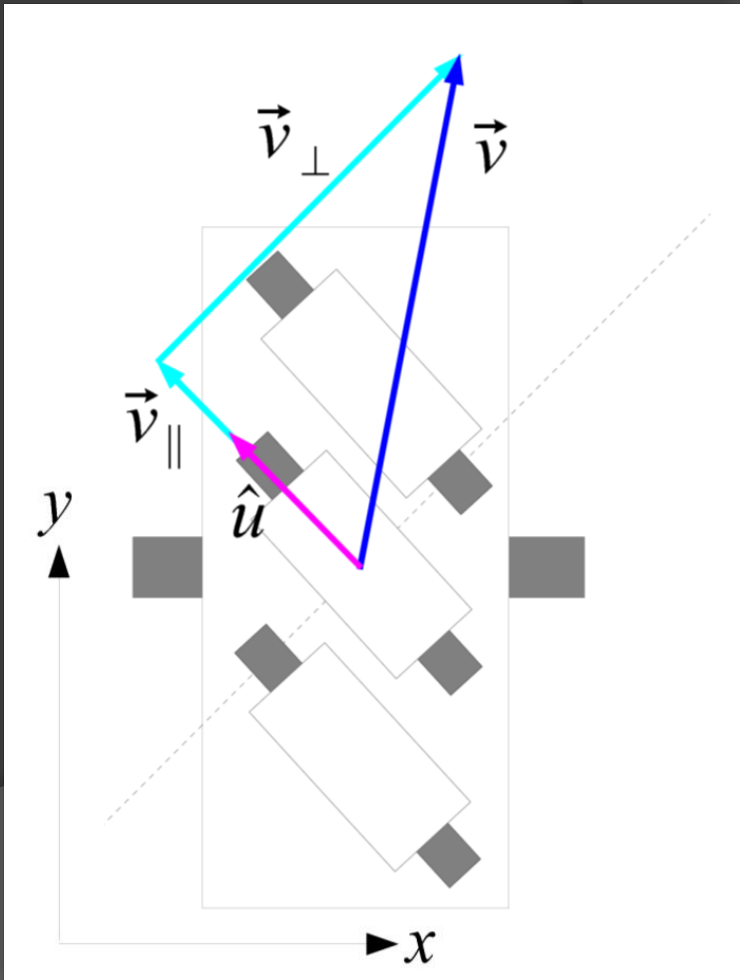
- Similar to omniwheel drive
- Conceptually: Resolve velocity into components parallel to wheel and parallel to roller
- Not easy to calculate directly (directions are not perpendicular), so do it in two steps:
  - Resolve to roller
  - Resolve to wheel



... resolve to Roller

- Resolve velocity into components parallel and perpendicular to roller axis
- $\hat{u}$ is <u>not</u> the same for each wheel; pick direction parallel to roller axis, in forwards direction
- Perpendicular component can be discarded
- $v_{\parallel} = \vec{v} \cdot \hat{u}$

$$= (v_x \hat{\imath} + v_y \hat{\jmath}) \cdot (-\frac{1}{\sqrt{2}} \hat{\imath} + \frac{1}{\sqrt{2}} \hat{\jmath})$$



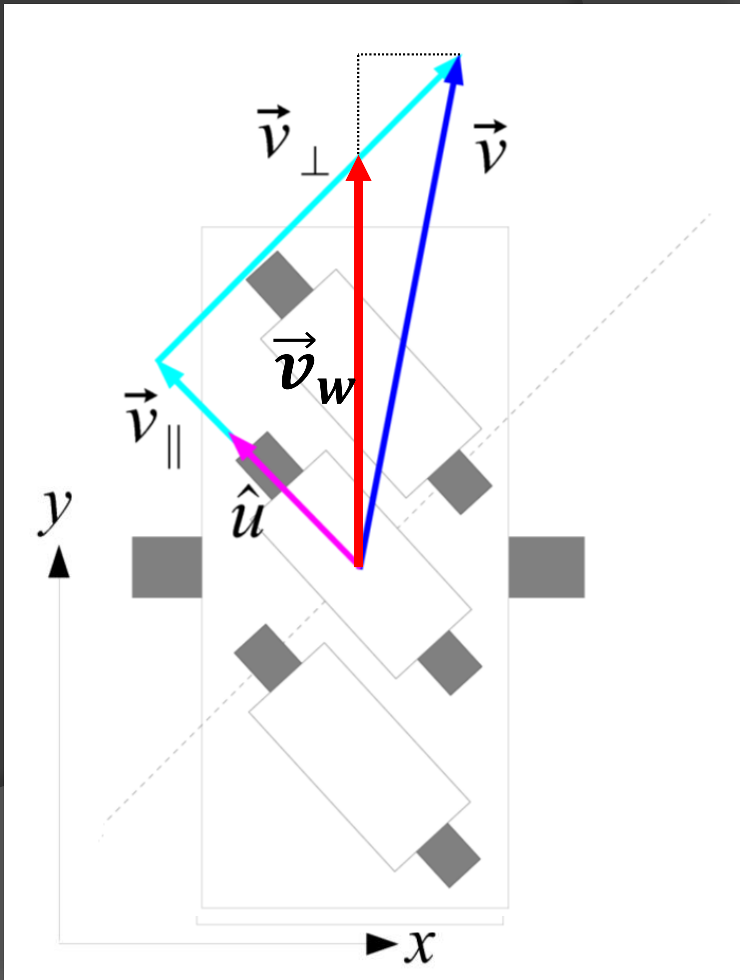
... resolve to Wheel

- Use component parallel to roller axis and resolve it into components parallel to wheel and parallel to roller
- v_w is the component parallel to the wheel
- When the angle is known, we can calculate v_w directly.
- E.g. for 45° inclination:

$$v_w = \frac{v_{\parallel}}{cos45°}$$

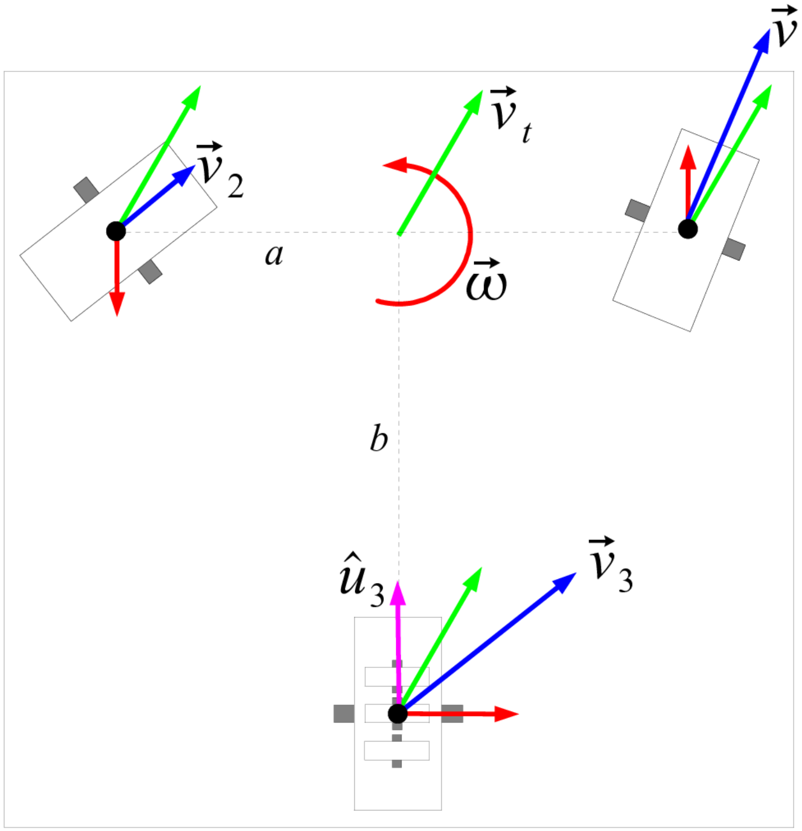
$$= \sqrt{2}\left(-\frac{1}{\sqrt{2}}v_x + \frac{1}{\sqrt{2}}v_y\right)$$

$$= -v_x + v_y$$



Hybrid conception

- Example: 2x swerve + 1x omniwheel:



$$
\begin{aligned}
v1_x &= v_{t_x} \\
v1_y &= v_{t_y} + \omega a \\
\\
v2_x &= v_{t_x} \\
v2_y &= v_{t_y} - \omega a \\
\\
v3_x &= v_{t_x} + \omega b \\
v3_y &= v_{t_y}
\end{aligned}
$$

$$
\begin{aligned}
v_{w1} &= \sqrt{v_{1_x}^2 + v_{1_y}^2} \\
&= \sqrt{v_{t_x}^2 + \left(v_{t_y} + \omega a\right)^2} \\
\theta_1 &= \text{arctan}\left(\frac{v_{1_y}}{v_{1_x}}\right) \\
&= \text{arctan}\left(\frac{v_{t_y} + \omega a}{v_{t_x}}\right) \\
\\
v_{w3} &= \vec{v}_3 \cdot \hat{u}_3 \\
&= \left(v_{3_x}\hat{\imath} + v_{3_y}\hat{\jmath}\right) \cdot \hat{\jmath} \\
&= v_{3_y} \\
&= v_{t_y}
\end{aligned}
$$