Denavit – Hartenberg

Rotate, move, move, rotate

Tohle mezi každým kloubem. Podle vlastností kloubu jsou některé ze čtveřice pevné konst. (například move může být furt stejný, pokud jsou klouby od sebe pevně na tyči)

Typical system composition:

chains composed of rotational and translational joint only.

joint k spojuje link k_{i-1} a k_i

link k spojuje jointy k_i a k_{i+1}

Vztah mezi LCS_{i-1} a LCS_i je tvořen:

1) rotací x_{i-1} osy okolo z_{i-1} o úhel γ_i
2) posun x_{i-1} osy směrem k z_{i-1} o vzdálenost d_i
3) posun LCS_{i-1} po ose x_i o vzdálenost a_i
4) rotace z_{i-1} osy okolo osy x_i o úhel α_i

1)
$$\begin{pmatrix} \cos(\gamma_i) & -\sin(\gamma_i) & 0 & 0 \\ \sin(\gamma_i) & \cos(\gamma_i) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

2)
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

3)
$$\begin{pmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

4)
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\alpha_i) & -\sin(\alpha_i) & 0 \\ 0 & \sin(\alpha_i) & \cos(\alpha_i) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

tedy jsou tam jen čtyři parametry: $\gamma_i, \alpha_i, d_i, a_i$

Všechno vynásobení pak dá:

$$A_{i-1}^{i} = \begin{pmatrix} \cos(\gamma_i) & -\sin(\gamma_i)\cos(\alpha_i) & \sin(\gamma_i)\sin(\alpha_i) & a_i\cos(\gamma_i) \\ \sin(\gamma_i) & \cos(\gamma_i)\cos(\alpha_i) & -\cos(\gamma_i)\sin(\alpha_i) & a_i\sin(\gamma_i) \\ 0 & \sin(\alpha_i) & \cos(\alpha_i) & d_i \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

DH system construction

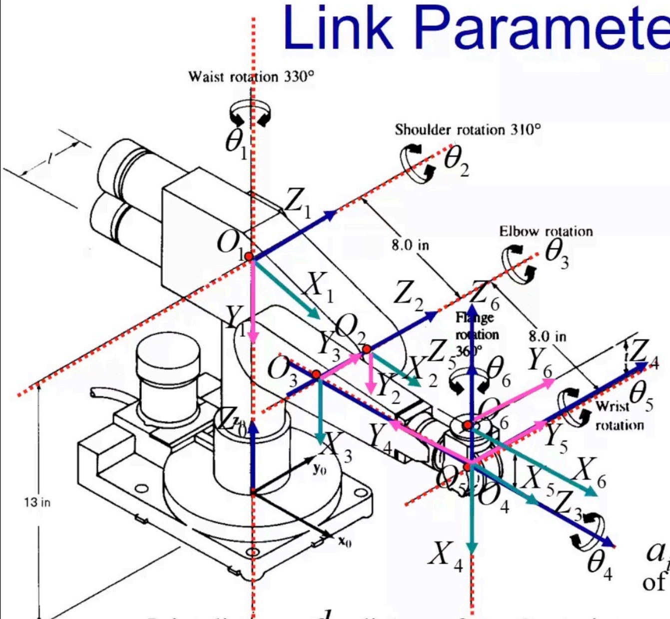
- Joints numbered 0..n (0 is the first, fixed, 1..n are the rest, moving)
- Links numbered 1..n (link i connects joints $i-1$ and i)
- Right-handed orthonormal coordinate system

- Let axis z_{i-1} be the axis of joint i movement, positive direction towards positive quadrant of the basic system
- Let axis x_i be perpendicular to z_{i-1} and z_i:
  - z_{i-1} and z_i identical – endpoint of joint i, parallel to x_{i-1}
  - skew – x_i share the normal z_{i-1} a z_i, positive direction from z_{i-1} towards z_i.
  - intersecting – x_i perpendicular to z_{i-1} and z_i, in the intersection, positive direction so that it moves along x_i from z_{i-1} to z_i in positive sense
- Set y_i axis to complete the right-handed orthonormal LCS_i
- Set LCS_i origin at intersection of z_{i-1} and z_i or (if they do not intersect) at intersection of their common normal and z_i
- Determine the four parameters:
  - ϑ_i … angle of rotation from x_{i-1} to x_i about z_{i-1}
  - d_i … distance from LCS_i origin to b_i along z_{i-1}, b_i is the intersection of x_i and z_{i-1} (or x_i and their common normal)
  - a_i … distance from b_i to LCS_i origin along x_i
  - α_i … angle of rotation from z_{i-1} to z_i about x_i

- z_n from the endpoint of last link either parallel to z_{n-1} or to some significant direction (e.g. supply cable)
- x_n from the endpoint of the last link so that it intersects z_{n-1}, positive direction towards the workspace.

Link Parameters

+ po směru ručiček
− proti



J	θ_i	α_i	a_i	d_i
1	θ_1	-90	0	13
2	θ_2	0	8	0
3	θ_3	90	0	-L
4	θ_4	-90	0	8
5	θ_5	90	0	0
6	θ_6	0	0	t

Na otáčení koukám ve směru osy x!

θ_i: angle from X_{i-1} to X_i about Z_{i-1}

α_i: angle from Z_{i-1} to Z_i about X_i

a_i: distance from intersection of Z_{i-1} & X_i to O_i along X_i

Joint distance d_i: distance from O_{i-1} to intersection of Z_{i-1} & X_i along Z_{i-1}

(courtesy EMU, Mustafa K. Uyguroğlu)

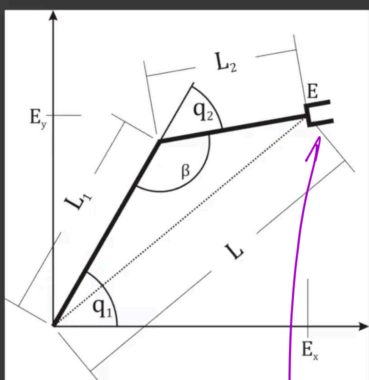
Inverse kinematics:

Jak nastavit klouby, abych se dostal do pozice?

Jednoduchý výpočet s ramenem.

Cosinova věta:

$$c^2 = a^2 + b^2 - 2ab\cos(\gamma)$$



$L^2 = L_1^2 + L_2^2 - 2\,L_1\,L_2\cos\beta$ → známe

$L^2 = E_x^2 + E_y^2$ → známe [x,y]

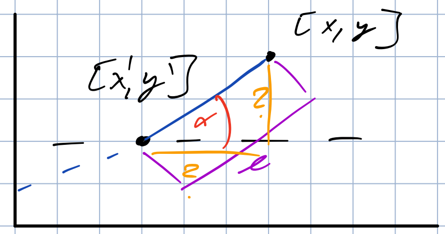
tedy i L

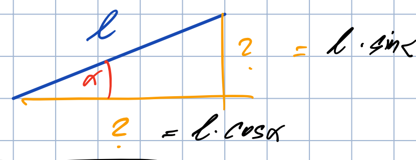
$\beta + q_2 = \frac{\pi}{2} \rightarrow \cos q_2 = -\cos\beta$

$q_2 = \arccos\frac{E_x{}^2 + E_y{}^2 - L_1^2 - L_2^2}{2\,L_1\,L_2}$

$q_1 = \mathrm{arctg}\frac{E_y}{E_x} + \mathrm{arctg}\frac{L_2\sin q_2}{L_1 + L_2\cos q_2}$

Tohle lze zjednodušit, jelikož pokud mám pevně zadaný úhel vůči base, tak umím určit předchozí joint:





$$\boxed{\begin{aligned} x' &= x - l \cdot \cos\alpha \\ y' &= y - l \cdot \sin\alpha \end{aligned}}$$