Computational complexity of NESH

1) Is NASH NP-Complete ?

No —> every game has at least one NASH

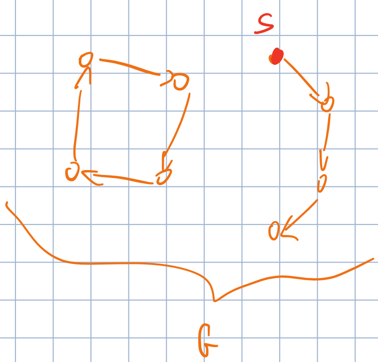
2) Is NASH FNP-Complete ? ~> istance of problem where for Yes, we require a solution

Probably no ↳ if NASH is FNP-complete, then NP = co-NP
↳ unlikely

End of the line problem:

Input: directed graph, where ∀ vertex has in- and out- degree ≤ 1, vertex s with in-degree 0 and out-degree 1

Output: vertex t ≠ s with in- or out-degree 0.



Problem: the graph is given by:

f: returning all neighbours in poly time

G can be exp. large

Class PPAD:

Problems that are reducable to the EOL problem in poly-time.

—> NASH belongs to it.

—> Sperners lemma belongs to it.

—> Brouwer's fixed point problem belongs to it.

—> Borsuk-Ulam problem belongs to it

NASH IS PPAD-complete.

↳ There is no polynomial general solution.

Other notions of equilibria

ε-Nash equilibria

$S = (s_1 \dots s_n)$ mixed strategy profile in $G = (P, A, u)$ is ε-Nash equilibrium

if: $\forall i \in P \ \forall s_i' \in S_i : \ u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i}) - \varepsilon \qquad (\varepsilon > 0)$

ε can be assigned value of our computation error (float64 error)

Advantages:

- always exists
  - every NE is surrounded by ε-NE.
- don't care about exact values

Disadvantages:

- ε-NE is not approximation of NE
  - alg. ε-NE might be far away from NE.
- not really efficiently computable
  - PTAS - alg. returning ε-approximate solution in time $O(n^{f(1/\varepsilon)})$
  - FPTAS - —//— in time $O((1/\varepsilon)^c n^d)$

Thm: $\forall G = (P, A, u)$ of 2 players, with $m = |A_1| = |A_2|$, payoff matrices having entries in $[0, 1]$, we can compute ε-NE in time $m^{O(\log(m)/\varepsilon^2)}$

Correlated equilibria:

In $G = (P, A, u)$ of n players, then probability distribution p on A $(p(a) \geq 0 \ \forall a \in A, \ \sum p(a) = 1)$ is correlated equilibrium if:

$$\forall i \in P, \ \forall a_i, a_i' \in A_i : \sum_{a_{-i} \in A_{-i}} u_i(a_i, a_{-i}) \cdot p(a_i, a_{-i}) \geq \sum_{a_{-i} \in A_{-i}} u_i(a_i', a_{-i}) \cdot p(a_i, a_{-i})$$

Disadvantages

- complicated definition

Assume there is a trusted 3rd party that gives recommendations to players what to play. p is publically known. If player i is recommended to play a_i, others stick with a_{-i} and he doesn't want to deviate from a_i, then we p is CE.

Advantages:

- always exists
- sometimes better than NE
- computable in poly-time

Prop: $\forall G=(P,A,u)$ $\forall NE$ $\exists$ corresponding CE

Pf: $s=(s_1 - s_n) \longrightarrow$ we define prob. dist. p_s on A by setting $p_s(a)=\prod_{i=1}^{n} s_i(a_i)$.

If s is NE, then p_s is CE.

Fix $i \in P$, $a_i, a_i' \in A_i$

if $s_i(a_i)>0$

$$u_i(a_i', s_{-i}) = \sum_{a_{-i}\in A_{-i}} u_i(a_i', a_{-i}) \prod_{j\neq i} s_j(a_j) = \frac{1}{s_i(a_i)} \sum_{a_{-i}\in A_{-i}} u_i(a_i', a_{-i}) \cdot p_s(a)$$

def of $u_i(a,s)$ def. of $p(s)$

$$u_i(a_i, s_{-i}) = \sum_{a_{-i}\in A_{-i}} u_i(a_i, a_{-i}) \prod_{j\neq i} s_j(a_j) = \frac{1}{s_i(a_i)} \sum_{a_{-i}\in A_i} u_i(a_i, a_{-i}) \cdot p_s(a)$$