Pure strategy of player i is an element $\prod_{h \in H_i} C_h$ = vector $(C_h)_{h \in H_i}$

Mixed strategy = prob. distr. on set of pure strategies of i.

Nash EQ = mixed strategy profile where strategies are best responses.

Extensive game G $\rightarrow$ normal form game G'

- by listing all pure strategies
- can be exponentially large

Behaviour strategy of player „i" = set of independent prob. dist. on C_h for the H_i

- $\beta(h)$
- in general mixed strategies and behaviour strategies may lead to different outcome.

Games of perfect recall where behaviour and mixed strategy coincide.

- let „i" be the player, t a node, sequence $\sigma_i(t)$ of i leading to t is a sequence of moves of „i" on the unique path leading to t.
- i has perfect recall if $\forall h \in H_i$ $\forall t, t' \in h : \sigma_i(t) = \sigma_i(t')$

  - we use σ_h = sequence leading to any node in h.
  - $\emptyset$ = empty sequence

In a game of perfect recall, every player has a perfect recall.

Kuhn's theorem:

In a game of perfect recall, every mixed strategy can be replaced by equivalent behaviour strategy and vice versa.

Ok: no proof

Sequence form of an extensive form game G

$= (P, S, u, C)$ where:

$P :=$ set of n players $(\{1 \ldots n\})$

$S := (S_1, \ldots, S_n)$ where $S_i := \{$sequences of player $i\}$

$S_i = \{\emptyset\} \cup \{\sigma_h c : h \in H_i, c \in C_h\}$

↳ sequence so far and one new move

$|S_i| = 1 + \sum_{h \in H_i} |C_h|$ ⟶ linear in the size of the tree of G

$u := (u_1 \ldots u_n)$ where $u_i : S \to \mathbb{R}$ where for $\sigma := (\sigma_1 \ldots \sigma_n)$

- can be described by high-dimensional matrices that are sparse.

we have $u_i(\sigma) = \begin{cases} u_i(\ell) & \text{if } \sigma \text{ leads to leaf } \ell. \\ 0 & \text{otherwise} \end{cases}$

⟵ if not leading to leaf.

C: describes behaviour strategies

- because of Kuhn theorem, also usable for NE
- will deal with behaviour strategies via realization plans

↳ $\beta_i = x : S_i \to \langle 0, 1 \rangle$

where $x(\sigma_i) = \prod_{c \in \sigma_i} \beta_i(c)$ → šance, že se ta kombinace celá stane.

→ realization prob.

- we will work with an equivalent definition of x:

$x : S_i \to \langle 0, 1 \rangle$ satisfying:

1) $x(\emptyset) = 1$

2) $\underbrace{x(\sigma_h)}_{\text{realiz. prob. of parent}} = \overbrace{\sum_{c \in C_h} x(\sigma_h c)}^{\text{sum of realiz. prob. of children}}$ for $\forall h \in H_i$

$= C_i$

$C := (C_1 \ldots C_n)$

Formally: x determines β_i on all relevant information sets h.

↳ $x(\sigma_h) > 0$

$$\beta_i(h, c) = \frac{x(\sigma_h c)}{x(\sigma_h)}$$

Computing NE in ZeroSumExtensiveFormGames

- $x = (x_\sigma)_{\sigma \in S_1} \in \mathbb{R}^{|S_1|}$
- $y = (y_\sigma)_{\sigma \in S_2} \in \mathbb{R}^{|S_2|}$
- define matrices $E \in \mathbb{R}^{(1+|H_1|) \times |S_1|}$

  $F \in \mathbb{R}^{(1+|H_2|) \times |S_2|}$

$$Ex = e = \left.\begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}\right\} 1+|H_1| \quad , \quad Fy = f = \left.\begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}\right\} 1+|H_2|$$

$$E = \begin{pmatrix} 1 & 0 & \dots & 0 \\ & C_1 & & \end{pmatrix} \qquad F = \begin{pmatrix} 1 & 0 & \dots & 0 \\ & C_2 & & \end{pmatrix}$$

Lets assume:

y = realization plan for player 2.

LP P for finding best response to y

$$\text{MAX } x^T A y \text{, where } A \in \mathbb{R}^{|S_1| \times |S_2|}, (A)_{ij} = \sum_t u_1(t) \cdot \beta_0(\sigma_0(t))$$

$Ex = e$

$x \geq 0$

t leaves where $\sigma_1(t) = i$

$\sigma_2(t) = j$

duality

D: $\text{MIN } e^T \cdot u$ $\quad \longrightarrow \quad$ D': $\text{MIN } e^T u$

$E^T u \geq Ay$ $\qquad\qquad E^T u - Ay \geq 0$

$u \in \mathbb{R}^{1+|H_1|}$ $\qquad\qquad Fy = f$

$\qquad\qquad\qquad\qquad y \geq 0$

Computing worse sequence scenario.

Analog. for the second player => computing NE.

Thm: NE in zero-sum extensive game of perfect recall are solutions of the LP D'.

Computing NE in 2-player extensive game of perfect recall.

- I can not use the final step of duality of creating D, D'.
- Can be solved by linear complementary programs.

(x, y) is NE $\iff \exists\, u, v$ s.t.:

to be equal to zero, for every x must be a complementary solution to v such that the $=0$ holds.

$$x^T(E^T u - Ay) = 0 \qquad y^T(F^T v - B^T x) = 0$$

$$Ex = e \qquad Fy = f$$

$$x \geq 0 \qquad y \geq 0$$

$$E^T u - Ay \geq 0 \qquad F^T v - B^T y \geq 0$$

x,y are realiz. plans

Can be solved by Lemke's algorithm.

At most in exponentional time.