Zápočet:

- všechny úkoly s nenulovým hodnocením

Celkový počet bodů má být:

3x úkol – 30b

1. quiz – 10b

2,3,4,5 quiz – 20b

prezentace – 15b

Test 1:

1) $x = (-2, -3, -4)$

$\|x\| = ?$

$$= \sqrt{(-2)^2 + (-3)^2 + (-4)^2} = \sqrt{4+9+16} = \sqrt{29} =$$

2) $x = (4, 4, c)$

$\|x\| = 8{,}3$

$c = ?$

$c \in \mathbb{R}$

$\|x\| = \sqrt{16 + 16 + c^2}$

$\|x\| = \sqrt{32 + c^2}$

$8{,}3 = \sqrt{32 + c^2}$

$68{,}89 = 32 + c^2$

$c^2 = 36{,}89$

$c = \pm 6{,}069596$

3) $x = (1, -1, 2, 1)$, $y = (-2, 2, -1, 1)$

$-2 + -2 - 2 + 1 = -5$

4) $x = (-2, 1, 2)$, $y = (-3, 2, 2)$

Určete první složku vektorového součinu

$w = x \times y = (x_2 y_3 - x_3 y_2, x_3 y_1 - x_1 y_3, x_1 y_2 - x_2 y_1)$

$$\begin{matrix} x_2 & & x_3 & & x_1 & & x_2 \\ & \times & & \times & & \times & \\ y_2 & & y_3 & & y_1 & & y_2 \end{matrix}$$

$2 - 4 = -2$

5)

$$\begin{array}{cc} & \begin{pmatrix} 1 & 3 \\ 4 & 2 \\ 2 & 1 \end{pmatrix} \\ \begin{pmatrix} 2 & 3 & 1 \\ 7 & 1 & 2 \end{pmatrix} & \begin{pmatrix} 16 & 13 \\ 15 & 25 \end{pmatrix} \end{array}$$

6)

$$\left(\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 2 & -1 & 1 & 1 \\ 1 & 0 & 3 & 4 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & 1 & 3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 4 & 4 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

7) $f(x) = \frac{1}{1+x^2}$, chceme derivaci v bodě 2,2. V bodě je funkce definovaná.

$$\left(\frac{1}{1+x^2}\right)' = \frac{0\cdot(\ldots) - 1\cdot(1+x^2)'}{(1+x^2)^2} = \frac{-1\cdot(0+2x)}{(1+x^2)^2} = \frac{-2x}{(1+x^2)^2} =$$

v bodě 2,2 : $-0{,}129011$

8) $f(x) = \frac{1}{1+e^{-x}}$ $\quad f'(x) = \frac{0\cdot(\ldots) - 1\cdot(1+e^{-x})'}{(1+e^{-x})^2} = \frac{e^{-x}}{(1+e^{-x})^2} = \frac{1}{(1+e^{-x})}\cdot\left(\frac{1+e^{-x}}{1+e^{-x}} - \frac{1}{1+e^{-x}}\right)$

10) $f(x) = \frac{1}{2}\sin x + \frac{x^4}{10}$ $\qquad x_0 = 2$

$f'(x) = \frac{1}{2}\cdot\cos x + \frac{4x^3}{10}$

$f''(x) = -\frac{1}{2}\sin x + \frac{12x^2}{10}$ $\nearrow$ pro $x=2$: $2{,}391275$

$T_x^f = f(x) + \frac{f'(x)}{1!}(x-a) + \frac{f''(x)}{2!}(x-a)^2$

$\left(-\frac{1}{2}\cdot\sin(-2) + 4{,}8\right)\cdot\frac{1}{2} =$

$-\frac{1}{4}\cdot\sin(-2) + 2{,}4 = 2{,}4087$

11) Spočtěte hodnotu integrálu:

$$\int_0^1 1+x^3\,dx = \int_0^1 1\,dx + \int_0^1 x^3\,dx = \Big[x\Big]_0^1 + \Big[\tfrac{1}{4}x^4\Big]_0^1$$

$$= 1 + \frac{1}{4} = \frac{5}{4} = 1{,}25$$

12) Parciální derivace:

$$f(x,y) = 2x^3y + \ln\frac{x}{2y}$$

13) $f(x,y) = (x-y-1)^2 - 2y^2$

Hodnota gradientu v bodě $(-1,1)$?

$$\frac{f(x,y)}{\partial x} = 2\cdot(x-y-1)\cdot\frac{\partial}{\partial x}(x-y-1) = 2(x-y-1)\cdot 1 \overset{(-1,1)}{=} -6$$

$$\frac{f(x,y)}{\partial y} = 2(x-y-1)\cdot\frac{\partial}{\partial y}(x-y-1) - 4y = 2(x-y-1)\cdot(-1) - 4y \overset{(-1,1)}{=} 2$$

1) $x = (-4, -3, 0)$, $\|x\| = ? = \sqrt{(-4)^2 + (-3)^2 + 0} = \sqrt{16 + 9 + 0} = \sqrt{25} = 5$

2) $x = (2, 1, c)$, $c \in \mathbb{R}$, $\|x\| = 10{,}1 = \sqrt{2^2 + 1^2 + c^2} = \sqrt{5 + c^2}$

$10{,}1 = \sqrt{5 + c^2} \to 102{,}01 = 5 + c^2 \to c^2 = 97{,}01 = 9{,}849365$

3) $x = (1, -3, 1, 1)$, $y = (-1, 2, 2, 1)$ Kolik je sk. součin?

$-1 - 6 + 2 + 1 = -4$

4) $x = (-2, 1, -2)$ $y = (-1, 2, 1)$ Vektorový součin?

$$\begin{array}{cccc} x_2 & x_3 & x_1 & x_2 \\ y_2 & y_3 & y_1 & y_2 \end{array}$$

$x_1 y_2 - x_2 y_1 = -4 + 1 = -3$

5) $A = \begin{pmatrix} 2 & 0 & 1 \\ 3 & 2 & 0 \\ 1 & 1 & 2 \end{pmatrix}$ $B = \begin{pmatrix} 4 & 0 & 2 \\ 1 & 2 & 3 \\ 0 & 3 & 1 \end{pmatrix}$

$AB = ?$

$$\begin{array}{cc} & \begin{pmatrix} 4 & 0 & 2 \\ 1 & 2 & 3 \\ 0 & 3 & 1 \end{pmatrix} \\ \begin{pmatrix} 2 & 0 & 1 \\ 3 & 2 & 0 \\ 1 & 1 & 2 \end{pmatrix} & \begin{pmatrix} 8 & 3 & 5 \\ 14 & 4 & 12 \\ 5 & 8 & 7 \end{pmatrix} \end{array}$$

6) $\left(\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 2 & -1 & 1 & 1 \\ 1 & 0 & 3 & 4 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & 1 & 3 \end{array}\right) \sim$

$\sim \left(\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & \cancel{4}\,1 & \cancel{4}\,1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right)$

$\sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right)$

7) $f(x) = \dfrac{1}{1+x^2}$ $f'(x) = \dfrac{0 \cdot (\ldots) - 1 \cdot (1+x^2)'}{(1+x^2)^2} = \dfrac{-2x}{(1+x^2)^2}$ v bodě $x = 0{,}6$

$= -0{,}6487889$

10) $f(x) = 2\cdot\cos x + x^4$

Taylorův polynom $T_x f: \sum_i \frac{f^{i}(x_0)}{i!}(x-x_0)^i$

$f'(x) = -2\cdot\sin x + 4x^3$

$f''(x) = -2\cdot\cos x + 12x^2$

$f'''(x) = 2\cdot\sin x + 24x$

$\frac{2\cdot\sin x + 24x}{3!} = -12{,}04704$

11) $\int_0^2 1 + x^3\,dx = \int_0^2 1\,dx + \int_0^2 x^3\,dx = [x]_0^2 + \left[\frac{1}{4}x^4\right]_0^2$

$= 2 + 4 = 6$

12)

13) $f(x,y) = (x-y-1)^2 - 2y^2$

Hodnota gradientu v bodě $(-1,1)$?

$\frac{f(x,y)}{\partial x} = 2\cdot(x-y-1)\cdot\frac{\partial}{\partial x}(x-y-1) = 2(x-y-1)\cdot 1 \overset{(1,0)}{=} 0$

$\frac{f(x,y)}{\partial y} = 2(x-y-1)\cdot\frac{\partial}{\partial y}(x-y-1) - 4y = 2(x-y-1)\cdot(-1) - 4y \overset{(1,0)}{=} 0$