

What is the entropy of a bigram distribution estimated from the following data, in the standard MLE way (start counting at position 2, i.e., e.g.,  $p_2(\text{so, similar})$  (joint prob.) =  $p(\text{so}) = 1/8$ ):

# No two numbers are so similar numbers .

Odpověď:

$$P(\text{no} | \#) = 1$$

$$P(\text{two} | \text{no}) = 1$$

$$P(\text{numbers} | \text{two}) = 1$$

$$P(\text{are} | \text{numbers}) = 0,5$$

$$P(\text{so} | \text{are}) = 1$$

$$P(\text{similar} | \text{so}) = 1$$

$$P(\text{numbers} | \text{similar}) = 1$$

$$P(\cdot | \text{numbers}) = 0,5$$

$$\text{Entropy} = -\sum p \cdot \log(p(x|y)) = -\left(1 \cdot 0 + \dots - \frac{1}{2} - \frac{1}{2}\right)$$

$\hookrightarrow \frac{1}{8}$

Conditional entropy has  $p(x) \cdot \log(p(x|y))$

! this can't be conditional !

Estimate a bigram LM in the standard MLE way from data T (collect counts starting at position 2):

# No two numbers are so similar numbers .

Then compute its cross-entropy against the following data S:

# No two numbers .

When computing the cross-entropy, also start at position 2.

Odpověď:

$$P_1(\text{no} | \#) = 1$$

$$P_1(\text{two} | \text{no}) = 1$$

$$P_1(\text{numbers} | \text{two}) = 1$$

$$P_1(\cdot | \text{numbers}) = 1/2$$

$$CE = -\sum p_1(x) \cdot \log p(x)$$

$$CE = -1 \cdot \frac{1}{2} = -\frac{1}{2}$$

Maximum Entropy principle:

- the most general distribution should win

- uniform gives the highest entropy  $\rightarrow$  therefore if I don't have data for  $P(X|Y, Z)$ , I will replace it with  $P(X)$ , which biases the later counts the least.

Estimate a trigram LM in the standard MLE way from data T (collect counts starting at position 3):

# # No two numbers are so similar numbers .

Then, compute its cross-entropy against the following data S:

# # No two numbers .

When computing the cross-entropy, also start at position 3.

Vyberte jednu z nabízených možností:

- a. infinity
- b. 0 ✗ No. It is true that no two trigrams overlap in data T, but it only means that entropy of  $p_3$  is zero. The cross-entropy depends on the data S and could be (very) different.
- c. 1/4 (0.25)
- d. 1

$$p(\text{no} \mid \#, \#) = 1$$

$$p(\text{two} \mid \#, \text{no}) = 1$$

$$p(\text{numbers} \mid \text{no}, \text{two}) = 1$$

$$p(\cdot \mid \text{two}, \text{numbers}) = 0$$

$$\log(0) = \infty,$$

Therefore the total sum will also be  $\infty$ .